# Bouncing Ball
# User's Guide

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# INTRODUCTION

Bouncing Ball is a program written for the Apple® Macintosh™ computer that simulates the dynamics of a ball bouncing on a sinusoidally oscillating table. Bouncing Ball 2.0 requires the use of a Macintosh with at least one megabyte (1024K) of memory, at least the 128K version of ROM (all Macintoshes except the original 128K Macintosh and the original Macintosh 512 have such a version), and the 6.0 System set (System 6.0 and Finder 6.1) or later. If you use Bouncing Ball with MultiFinder™, please read "MultiFinder and Memory Considerations" in the section "Tips and Techniques."

This manual explains how to use Bouncing Ball. Before you start, you should already know how to perform basic operations with your Macintosh. You should know how to use the Finder™ and the mouse, manipulate windows, scroll, pull down menus, and choose commands. If you need information about any of these topics, consult your owner's guide.

This manual has six main sections:

- Getting Started
- Types of Simulation
- Bouncing Ball Windows
- Bouncing Ball Menus
- Tips and Techniques
- Selected Error Messages

**Getting Started** shows you how to create simulations and describes the sample files that accompany Bouncing Ball.

**Types of Simulation** describes the three different types of simulations Bouncing Ball can run.

**Bouncing Ball Windows** describes the windows Bouncing Ball uses to display data from the various simulations.

**Bouncing Ball Menus** contains a directory of all Bouncing Ball menu commands, arranged in the order they appear on the pull-down menus.

**Tips and Techniques** describes various ways to tailor your use of Bouncing Ball. It includes a discussion of the memory requirements of Bouncing Ball.

**Selected Error Messages** lists the error messages that Bouncing Ball generates and describes what to do when they appear.

# GETTING STARTED

1. Start the Macintosh by turning it on. If you do not have a hard disk, insert a Macintosh system disk.
2. Insert the Bouncing Ball disk and open it, if it is not already open.
3. Open the Bouncing Ball folder.
4. Open the Bouncing Ball application.
5. After the program starts up, choose **Initial Conditions** from the **Control** menu. Change the initial ball velocity and phase (phase is measured in relation to the table, originally from 0 to 1, 1 corresponding to $2\pi$ radians) to some other values, or leave them the same, then click the OK button.

The simulation should now be running. Note that there are four windows currently displayed; additional windows can be activated through the **Windows** menu. The upper left window, *Trajectory*, contains a graph of the motion of the ball and table, the horizontal axis representing time. The lower left window, *Animation*, contains an animation of the ball and table. The upper right window, *Untitled* (when you open a simulation or save one, this window takes on the name of the file; it will often be called the *Impact Data* window), lists the table parameters (amplitude, frequency, and damping) and, for each collision between the ball and table, the sequential number of the collision from the beginning of the simulation, the ball's velocity after impact, the phase (again, from 0 to 1), and the total simulation time elapsed. The lower right window, *Impact Map*, plots, for each collision, the ball's outgoing velocity versus the phase.

In addition to the initial velocity and phase, you can control parameters relating to the table—amplitude, frequency, and damping—by choosing **Table Parameters** from the **Control** menu.

For instance, to illustrate the period-doubling route to chaos in the bouncing ball system, we increase the table amplitude by small increments in successive simulations. First reset the initial conditions to the original values, velocity = 10 cm/s, phase = 0. The default values for the table parameters, which lead to a period one orbit, are 0.01 cm for the amplitude, 60 Hz for the frequency, and 0.5 for the damping coefficient. To generate a period two orbit, increase the table amplitude to 0.011 cm by choosing **Table Parameters** from the **Control** menu and changing the table amplitude from 0.01 to 0.011. Now click OK to run the simulation with this new parameter value. Follow the same procedure to increase the table amplitude to 0.012 to produce a chaotic trajectory.

An alternate way to choose new initial conditions is to click in the body of the impact map; the **Initial Conditions** dialog box will come up with velocity and phase corresponding to the point of the mouse-click. In addition, clicking in certain areas of the various windows calls forth appropriate dialog boxes. These active areas are known as "hot spots." Such hot spots are located along the two axes of the *Impact Map* window. The section "Bouncing Ball Windows" discusses these areas further.

Now we'll look at Bouncing Ball's two other types of simulation, both of which are based on the simulation described above.

## Bifurcation Diagram

To create a *bifurcation diagram*, follow these steps (the first three steps simply restore initial values):

1. Use the **Table Parameters** command to restore the initial values, amplitude of 0.01 cm, frequency of 60 Hz, and damping coefficient of 0.5.
2. Use the **Initial Conditions** command to restore the original initial conditions, velocity of 10 cm/s, phase of 0.
3. Clear the current data by choosing **New** from the **File** menu.
4. Open the *Bifurcation Diagram* window by choosing **Bifurcation Diagram** from the **Windows** menu.
5. Choose **Bifurcation Diagram** from the **Special** menu and click OK.

The program will generate a diagram showing how the solutions to the Bouncing Ball system vary as the table's amplitude varies. Specifically, you'll see how the phase (vertical axis) changes with the table amplitude (horizontal axis). The bifurcation diagram stops automatically after it has moved through the specified amplitude range.

## Basins of Attraction

To create a *basins of attraction* diagram, follow these steps (again, the first three steps simply restore the program to how it would be upon start-up):

1. Use the **Table Parameters** command to restore the initial values, amplitude of 0.01 cm, frequency of 60 Hz, and damping coefficient of 0.5.
2. Use the **Initial Conditions** command to restore the original initial conditions, velocity of 10 cm/s, phase of 0.
3. Clear the current data by choosing **New** from the **File** menu.
4. Open the *Basins of Attraction* window by choosing **Basins of Attraction** from the **Windows** menu.
5. Choose **Basins of Attraction** from the **Special** menu, input "8" in the Increment Size (or "step size") field, and click OK.

The program will generate a diagram showing which initial conditions eventually go to which types of solutions. Each different type of solution (sticking, period one, chaotic—really "other"—etc.) is represented by a different pattern or color in the basins of attraction chart, which is closely related to the impact map chart. A step size of 8 is used so that the diagram will be completed relatively quickly. The basins of attraction diagram stops automatically when it has discovered the eventual type of solution for all initial conditions in the charted range.

## Sample Files

To help you quickly see some of its capabilities, Bouncing Ball is packaged with the following sample files. You can either double-click on these sample files or open them with the **Open** command in the **File** menu.

| | |
|---|---|
| **Period 2** | A simulation that converges to a period two solution |
| **Ball Sticks** | A simulation in which, after several iterations, the ball becomes stuck to the table |
| **Strange Attractor (lengthy)** | An example of chaos (it contains 500 iterations, so it takes a while to open, especially on slower Macintoshes) |
| **Sample Bifurcation Diagram** | A bifurcation diagram in which the amplitude was incremented from 0.01 cm to 0.012 cm |
| **Sample Basins of Attraction** | A basins of attraction diagram, with velocity charted from –3.77 to 10 and a step size of 2 |
| **Complex Basins of Attraction** | A basins of attraction diagram, with table amplitude 0.012, frequency 70, damping 0.5, velocity charted from –5.28 to 15, and a step size of 1. This diagram has basins for both a period one solution and a period three solution. |

# Types of Simulation

Bouncing Ball can perform three types of simulation. The central simulation to Bouncing Ball is the straightforward one described in the beginning of the section “Getting Started,” wherein you set values for the table parameters and the initial conditions, then watch the ball bouncing on the table in any combination of seven windows. This type of simulation will be called the “base” simulation.

The other two types of simulation, referred to as “special” simulations, are based on the base simulation. Each special simulation is displayed in its own window. The two types of special simulations are the bifurcation diagram and the basins of attraction diagram.

## Base Simulation

The base simulation represents a computer version of the Bouncing Ball experiment. Using the **Initial Conditions** and **Table Parameters** menu commands, you set the experimental parameters of the experiment, then run the simulation.

As the simulation progresses, you can watch the data displayed in the *Trajectory*, *Animation*, *Impact Data* (“*Untitled*” at the beginning), *Impact Map*, *Phase Space*, *Impact Phases*, and *Time Series* windows. The section “Bouncing Ball Windows” describes how each of these windows displays the data.

Typical parameters for a base simulation are listed below. Note that in the Bouncing Ball program all measurements are in cgs (centimeters-grams-seconds) units.

| *Parameter* | *Typical experimental range* |
|---|---|
| Table’s amplitude | 0–0.1 cm |
| Table’s frequency | 0–100 Hz |
| Damping coefficient | 0–1 |
| Initial velocity | 0–100 cm/s |
| Initial phase | 0–1 |

## Bifurcation Diagram

In the *bifurcation diagram*, you select a *range* of values for one table parameter, either the frequency or the amplitude. The program then starts the Bouncing Ball system at the beginning of the table parameter range. It lets the base simulation run for a certain number of iterations (to eliminate the “transient” solution), then charts an impact value (phase or velocity) on the vertical axis versus the table parameter on the horizontal axis. It then increments the table parameter (by a value corresponding to one screen pixel) and repeats the process, until it reaches the end of the parameter range.

*WARNING*: The bifurcation diagram runs very slowly, especially on slow Macintoshes.

What one finds in the bifurcation diagram is that, as the table parameter is changed, the type of solution the Bouncing Ball system reaches can change. For instance, in the amplitude range from 0.01 to 0.012 cm, with frequency 60 Hz and a damping coefficient of 0.5, and initial conditions of velocity 10 cm/s and phase 0, the system will go through a period one solution, a period two solution, a period four solution, a period eight solution, and so on until it reaches chaos.

To create a bifurcation diagram, choose **Bifurcation Diagram** from the **Special** menu and complete the dialog box as described below.

1. Select the table parameter you want to increment: amplitude or frequency.
2. Specify the range of values for the parameter chosen in step 1.
3. Select the impact measurement you want to chart: phase or velocity.
4. Specify the range of displayed values for the impact measurement selected in step 3.
5. Specify the values for the remaining table parameters.
6. Specify the initial conditions for the first iteration.
7. Specify the number of iterations you want the simulation to run before charting the data, for each value of the table parameter. Checking the Auto box puts in the word "Auto" and gives a value based on the damping coefficient—the higher the damping coefficient, the longer it typically takes to work through the transient solution. In addition, with Auto checked, if after reaching the automatic value Bouncing Ball determines that it has not reached periodicity, but that it will probably reach periodicity if continued, it will increase this number of initial iterations. Checking Auto tends to give the most accurate results. *Note*: If Bouncing Ball determines that periodicity has been reached before reaching the given number of iterations, it will automatically start charting the data.
8. Specify the number of iterations you want the simulation to chart. Checking the Auto box puts in the word "Auto." If you choose the automatic option, Bouncing Ball charts several iterations, then charts an additional number of iterations based on the dispersion of those first charted iterations. If, because of a periodic solution, Bouncing Ball decides it does not need to display any more solutions, it stops. *Note*: In a chaotic system, in full-screen view on a small Macintosh, the automatic value described above can easily lead to 500 or more iterations, which can be quite time consuming. A value of 100 or so should lead to satisfactory results in most circumstances.

9. Select the window size desired: if As-Is is selected, the size of the bifurcation diagram will be such that it will fit into the *Bifurcation Diagram* window as currently sized; if Full Screen is checked, the bifurcation diagram will take up almost the whole screen (it will just fit into a window that has been "zoomed out"). *Note*: The status of these radio buttons will not physically change the size of the *Bifurcation Diagram* window, but it will affect how the scroll bars of that window, if any, operate.

10. Set these miscellaneous bifurcation diagram parameters:

    a. Graph in All Windows—As remarked above, the bifurcation diagram actually creates a series of base simulations. Check this box if you wish to display the base simulations in all appropriate windows, as described in the "Base Simulation" part of this section. This allows you to see in detail what's happening at each value of the table parameter; however, it greatly slows down the creation of the bifurcation diagram.

    b. (Re)Start—Check if you want to start the bifurcation diagram after clicking OK.

    c. Use Color—Check if you want the bifurcation diagram to display in color. When displayed in color, each type of solution (refer to the "Types of Solutions" discussion of this section for a description of the different solution types) displays in a different color. If you do not have a color monitor, or if it is not set for 4- or more bit color (16 or more colors), this button will be disabled (grayed). *Note*: If your monitor is set for gray-scale, for 4 or more bits, you can implement "color"—it will simply use different shades of gray.

11. If color is being used, pressing the Colors button will bring up the same dialog box as the **Colors** command from the **Special** menu.

12. Press OK to complete the input, Cancel to cancel it.

## Basins of Attraction

In the *basins of attraction* diagram you select a *range* of initial conditions to try. The program then runs a base simulation for the first initial condition in the range, looks to see what type of solution (attractor) that initial condition converges to, and graphically marks that initial condition with a pattern or color to depict it as belonging to the found attractor. The "Types of Solutions" discussion that follows describes the different types of solutions that Bouncing Ball recognizes. It then increments the initial conditions and repeats the process, until it has covered all initial conditions in the selected range. The basins of attraction chart is thus very similar to the impact map, in that both chart impact phases and velocities.

Until it finds the type of solution, Bouncing Ball plots and then erases data points as they are created. When it does find the solution type, Bouncing Ball then gives the initial condition point the appropriate pattern or color. If the "faster" option is on (see below), it will also give the appropriate pattern or color to all the data points it found along the way.

*WARNING*: Like the bifurcation diagram, the basins of attraction diagram runs very slowly, especially on slow Macintoshes.

One finds in the basins of attraction diagram exactly which initial conditions and regions of initial conditions converge to which types of solutions.

To create a basins of attraction diagram, choose **Basins of Attraction** from the **Special** menu and complete the dialog box by inputting the information listed below.

1. Specify the range of values over which you want to increment the phase.
2. Specify the range of values over which you want to increment the velocity.
3. Specify the fixed table parameter values.
4. Specify the maximum number of iterations you want the simulation to create at each initial condition. Checking the Auto box puts in the word "Auto" and yields a value based on the damping coefficient; the higher the damping coefficient, the longer it typically takes to work through the transient solution. Also, with Auto checked, if after reaching that number of iterations Bouncing Ball determines that it has not reached periodicity, but that it will probably reach periodicity if continued, it will increase this number of iterations. Checking Auto tends to give the most accurate results. If after this number of iterations Bouncing Ball has found no periodic solution, it assumes that it is a chaotic (or "other") solution.
5. Specify the "step size," the size of the velocity and phase increments, in screen pixels. The larger the value (limited to the range 1-8), the faster the diagram will be completed, but the lower the resolution.
6. Select the window size desired: if As-Is is checked, the size of the basins of attraction will be such that it will fit into the *Basins of Attraction* window as currently sized; if Full Screen is checked, the basins of attraction will take up almost the whole screen (it will be such that it will just fit into a window that has been "zoomed out"). *Note*: The status of these buttons will not physically change the size of the *Basins of Attraction* window; it will affect how the scroll bars of that window, if any, operate.
7. Set these miscellaneous basins of attraction parameters:
   a. Faster—If this is checked, Bouncing Ball will remember the coordinates of each impact as it goes. Then, when the solution type is found, it will mark all of the impact values found along the way as initial conditions belonging to the basin of the solution type found. Because all these impacts are being collected, succeeding iterations can look to see if they are already in a basin; if several

impacts in a row are all in the same basin, Bouncing Ball assumes that the current iteration is converging to the same solution as the initial conditions in that basin. Without this option, only the true initial condition is marked. Because of rounding, using this option gives less accurate results than the alternative, and one will receive results that are clearly off in places. However, this option speeds up the chart tremendously.

b. Graph in All Windows—As remarked above, the basins of attraction diagram actually creates a series of base simulations. Check this box if you wish to display the base simulations in all appropriate windows, as described in the "Base Simulation" part of this section. This allows you to see in detail what's happening at each set of initial conditions; however, it greatly slows down the creation of the basins of attraction.

c. (Re)Start—Check if you want to start the basins of attraction diagram after clicking OK.

d. Use Color—Check if you want the basins of attraction to display in color. When displayed in color, each type of solution (sticking, period one, other (chaotic), etc.) displays in a different color; otherwise each type of solution is displayed in a different (gray) pattern. If you do not have a color monitor, or if it is not set for 4- or more bit color (16 or more colors), this button will be disabled (grayed). *Note*: If your monitor is set for gray-scale, for 4 or more bits, you can implement "color"—it will simply use different shades of gray. Also, Bouncing Ball does allow you to go freely back and forth between color display and pattern display; a basins of attraction diagram created in black and white can later be displayed in color, without loss of resolution.

   Note when patterns are being used: Because of the difficulties inherent in "coloring" different regions (using gray patterns) on a black and white monitor, all initial conditions corresponding to the same solution type may not be marked in the same way. For instance, with a "step size" of 2, each point is really a $2 \times 2$ rectangle of pixels. For points corresponding to a solution represented by the darkest gray pattern, some $2 \times 2$ pixel blocks will be all black, some partially white. However, the entire *region* corresponding to this solution type would accurately show the dark gray pattern. This implies that, when patterns are being used, it is not always possible to tell which specific pixel blocks correspond to which solutions.

8. This button will be entitled "Colors..." or "Patterns...", depending on whether color is being used. Pressing it will bring up the same dialog box as the **Colors** command or the **Patterns** command, whichever is appropriate, from the **Special** menu.

9. Press OK to complete the input, Cancel to cancel it.

## Types of Solutions

In a basins of attraction or bifurcation diagram, Bouncing Ball recognizes the following types of solutions:

- **Sticking**
- **Period One**
- **Period Two**
- **Period Three**
- **Period Four**
- **Period Eight**
- **Other**
- **Unphysical**

"Sticking" solutions are solutions for which the ball eventually comes to rest with respect to the table, or "sticks" to it. "Period One" through "Period Eight" solutions are just what their names suggest; periodic solutions of the given periods. "Other" solutions, as you might guess, are those that do not fit into any other category. In practice, they are most often chaotic solutions, but can also be other $2^n$-periodic solutions, such as period sixteen, or still other periodic solutions, such as period five. In addition, slow convergence and insufficient iterations can cause Bouncing Ball to think that, say, a period two solution is an "other" solution. "Unphysical" solutions, on the other hand, are those that represent physical impossibilities; in this case, these are initial conditions wherein the initial ball velocity is less than the initial table velocity, meaning the ball would move through the table (remember, the ball is initially on the table).

# BOUNCING BALL WINDOWS

This section describes the nine windows Bouncing Ball uses to display data.

In all windows, any measurements shown are in cgs units.

## Trajectory

The *Trajectory* window shows the position of the ball and the table as a function of time. It thus contains two traces. One trace represents the motion of the table and is sinusoidal. The other represents the motion of the ball and consists of a series of parabolas. The bottom of the window displays the initial conditions.

The scaling in this window depends on values entered with the **Trajectory Scaling** command from the **View** menu. You set the number of table periods that would be shown horizontally if the *Trajectory* window were at its largest size (zoomed out) and the number of table amplitudes that would be shown vertically under the same circumstances. Tick marks on the vertical axis are set at every two table amplitudes.

When the traces reach the right edge of the graph, by default the graph is cleared and the traces continue from the left edge. If you prefer, the traces can instead smoothly scroll to the left as data is added to the right. Use the **Scroll Trajectory** command in the **View** menu to toggle this scrolling. (Technically, scrolling can be made the default with the **Save Defaults** command.) Scrolling the traces takes considerably more time.

## Impact Data (“Untitled”)

This text window contains, for each impact, the sequential number of the impact, the outgoing ball velocity, the table phase, and the total time the simulation has run. Bouncing Ball provides no way to control the precision used to display the numbers in this window.

When a base simulation is running, text is being continuously created, so Bouncing Ball does not allow you to scroll through or select text. When the iteration is stopped, however, you may scroll through the text, make selections, and copy information to the Clipboard.

When a simulation is saved or opened, this window takes on the name of the corresponding file.

When approximately 32K of text is created (approximately 850 solutions), most of the text is eliminated from the window (but still stored and able to be saved), and an appropriate message appears at the beginning of the text. The preceding text may be viewed by playing back the solutions (**Play Back Solutions** from the **Control** menu) or by saving it and viewing the saved (text) file in a word processor or other application.

## Impact Map

In the impact map, Bouncing Ball plots the outgoing ball velocity versus the table phase for each impact. The map displays the entire phase range on the horizontal axis, unless the plot is magnified. Vertically, the minimum velocity is equal to the minimum table velocity. The maximum velocity is set by default to either a "trapping region" value, or the initial ball velocity that would just barely take the ball's trace from the maximum table amplitude to the top of the *Trajectory* window in its largest position, whichever is smaller. This maximum velocity can be adjusted with the **Max. Velocity** command. Any of these values may be changed (and later restored) by magnifying the impact map, as described below and in the description of the **Impact Map Magnification** command.

*Special Features*: Clicking inside the impact map brings up the **Initial Conditions** dialog box with values corresponding to the point of the click. Dragging inside the impact map brings up the **Impact Map Magnification** dialog box, with values corresponding to the dragging endpoints. Hold the Shift key down to bypass the dialog box.

## Animation

The ball and the table are seen in motion in this window. The animation is useful in visualizing the system, although it pauses slightly at each impact due to computational demands. The animation is somewhat smoother when playing back solutions than when generating them in the first place. It can also be made smoother by slowing down the simulation, with the **Simulation Speed** command.

The vertical scaling is the same as that used in the *Trajectory* window.

## Phase Space

As the ball moves, this window displays its height versus its velocity. Note that the phase space is graphed only when this window is open.

The vertical scale is the same as that used in the *Trajectory* window. The horizontal scale is such that, when the window is at its largest size (zoomed out), it displays from the negative of the maximum velocity to the maximum velocity, with the maximum velocity determined as in the *Impact Map* window. For instance, if the maximum velocity has been set to 10, the horizontal axis will display from –10 to 10, when the window is at its largest size. Note in particular that, if the window is not at its largest horizontal size, it will display a smaller velocity range.

## Impact Phases

This window contains an "embedded phase space," and looks and acts somewhat like the *Impact Map* window. It plots the phase of the current impact on the vertical axis versus the phase of a previous impact. Initially Bouncing Ball looks at the immediately preceding impact; the **Impact Phases Offset** command (**View** menu) allows you to specify which prior impact will be charted.

Unless magnified, the impact phases chart displays the entire phase range, as specified by the **Phase Range** command, on both axes.

*Special Feature*: Dragging inside the impact phases chart brings up the **Impact Phases Magnification** dialog box, with values corresponding to the dragging endpoints. Hold the Shift key down to bypass the dialog box.

## Time Series

This window generates simulation data sampled at a user-specified number of times per table period. It generates the simulation time, the ball velocity, and the ball position above the lowest table position, as graphed in the *Trajectory* and *Phase Space* windows. You can set the sampling rate with the **Time Series** command from the **View** menu. Bouncing Ball provides no way to control the precision used to display the numbers in this window.

Generating time series data slows up the simulation far more than any other display of the data. For this reason, you must tell Bouncing Ball explicitly to begin generating the data, usually with the **Use Time Series** command from the **View** menu, and Bouncing Ball will guess when you wish to stop generating the data. There are thus several dialog boxes that arise in conjunction with this window.

This window acts the same as the *Impact Data* window in terms of scrolling, selection, and behavior when the amount of text approaches 32K.

## Bifurcation Diagram

Bouncing Ball uses this window to create and display bifurcation diagrams, as discussed in the section “Types of Simulation.” The bottom of the window displays the fixed bifurcation diagram parameters, as set in the **Bifurcation Diagram** dialog box, and the initial conditions and the value of the table parameter being used for the latest iteration.

When a bifurcation diagram is saved or opened, this window takes on the name of the corresponding file.

*Special Features*: Clicking inside the body of the bifurcation diagram brings up the **Bifurcation Diagram** dialog box. Dragging inside the bifurcation diagram brings up the same dialog box, with range values corresponding to the dragging endpoints (which effectively allows magnification, although, since it cannot magnify existing data, the data must be regenerated). Hold the Shift key down to bypass the dialog box and create the bifurcation diagram for the magnified area.

## Basins of Attraction

Bouncing Ball uses this window to create and display basins of attraction diagrams, as discussed in the section “Types of Simulation.” The bottom of the window displays the fixed basins of attraction parameters, as set in the **Basins of Attraction** dialog box and the latest initial conditions.

When a basins of attraction diagram is saved or opened, this window takes on the name of the corresponding file.

*Special Features*: Clicking inside the body of the basins of attraction diagram brings up the **Basins of Attraction** dialog box. Dragging inside the basins of attraction diagram brings up the same dialog box, with range values corresponding to the dragging endpoints (which effectively allows magnification, although, since it cannot magnify existing data, the data must be regenerated). Hold the Shift key down to bypass the dialog box and create the basins of attraction diagram for the magnified area.

## Hot Spots

Hot spots are areas in Bouncing Ball windows that may be clicked to execute specific commands. Below is a summary of the hot spots in each window. (Clicking/dragging in the graphing regions of some windows has other effects; see the "Special Features" parts of the window-by-window discussion above.)

| Window | Hot spot region | Command |
|---|---|---|
| *Trajectory* | Horizontal axis | **Initial Conditions** |
| | Vertical axis | **Trajectory Scaling** |
| *Impact Data* | Header area | **Table Parameters** |
| *Impact Map* | Horizontal axis | **Phase Range** |
| | Vertical axis | **Max. Velocity** |
| *Animation** | Ball | **Initial Conditions** |
| | Table | **Table Parameters** |
| *Phase Space* | Horizontal axis | **Max. Velocity** |
| | Vertical axis | **Trajectory Scaling** |
| *Impact Phases* | Horizontal axis | **Phase Range** |
| | Vertical axis | **Impact Phases Offset** |
| *Time Series* | Header area | **Time Series** |
| *Bifurcation Diagram* | Horizontal axis | **Bifurcation Diagram** |
| | Vertical axis | **Bif. Diagram Phase Range** (if command is active) or **Bifurcation Diagram** |
| | Whole window (when blank) | **Bifurcation Diagram** |
| *Basins of Attraction* | Horizontal axis | **Basins of Attr. Phase Range** (if command is active) or **Basins of Attraction** |
| | Vertical axis | **Basins of Attraction** |
| | Whole window (when blank) | **Basins of Attraction** |

* The *Animation* window hot spots only function "during" an impact; the hot spots are located where the ball and table collide.

# Bouncing Ball Menus

In addition to the requisite **Apple** menu, Bouncing Ball has seven menus. The **File** menu contains the standard commands for file manipulation, printing, and quitting. The **Edit** menu contains the standard Macintosh editing commands (though Bouncing Ball only implements the **Copy** command; the rest are supplied for desk accessory support), and a few commands best described as preferences commands. The **Control** menu contains the commands that allow you to control Bouncing Ball's simulation: starting/stopping it, setting system parameters, and so on. The **View** menu allows you to control how you view the simulation; it contains commands to set scaling, hide old data, and so on. The **Special** menu contains the commands that control Bouncing Ball's special types of simulation, bifurcation diagrams and basins of attraction. The **Sound** menu allows you to control Bouncing Ball's sound. The **Windows** menu has commands to control window size and placement and to open windows.

The remainder of this section describes each of Bouncing Ball's menu commands.

## File Menu

**New**

The **New** command stops the current simulation and clears the data from all windows. The **New** command closes the current file(s), if any.

**Open**

The **Open** command opens a simulation stored on the disk and displays it on the screen.

Since Bouncing Ball files (from base simulations) are stored as text files, the **Open** dialog box will display all text files. Opening a text file not in Bouncing Ball format will cause an error message and will clear all the windows. Similarly, Bouncing Ball cannot open its own time series files, so if you try to open a time series file, an error message will follow. See the **Save** command for more information on how Bouncing Ball saves simulations.

Check the Graph while opening (if appropriate) box to graph data in all applicable windows while opening. This option only works when a base simulation is being opened. If this box is not checked, data from a base simulation will be displayed in the *Impact Data*, *Impact Map*, and *Impact Phases* windows only. The **Save Defaults** command remembers the current setting of this command as the default setting.

Command-period will abort the opening of a base simulation file.

Opening Bouncing Ball files is somewhat different from standard Macintosh opening. Usually, when one opens a Macintosh file, it appears in *its own* window. Bouncing Ball files open into *pre-existing* windows. With the exception of size and position, these windows maintain their attributes when a new file is opened; think of these windows as tools to inspect whatever data is displayed, tools that are not reset for new data.

**Save**

The **Save** command saves the current simulation on the disk from which it was opened (or to which it was "saved as"). Bouncing Ball saves the simulation with the name used the last time you saved it, and replaces the old copy of the simulation on the disk. You can only use the **Save** command with simulations that you have previously saved with the **Save As** command; if the simulation has not previously been saved, this command acts like the **Save As** command.

Bouncing Ball decides what to save by looking at the active window. If the active window is the *Time Series* window, it saves the time series data. If the front window is the *Bifurcation Diagram* or *Basins of Attraction* window, and there is a current bifurcation diagram or basins of attraction diagram, respectively, it saves that data. In all other cases, it saves the base simulation data, the data that appears in the *Impact Data* window.

Bouncing Ball saves base simulations and time series data as text files, just as the simulation appears in the corresponding window (including, of course, any data that has scrolled out of the window). The data can thus be easily passed to other applications for further processing. Bouncing Ball also saves appropriate window sizes and positions, in such a way that these will not interfere with the transfer to other applications (this window information is stored in the resource fork of the file). Note that, except for window sizes and positions, only the values appearing in the text window (table parameters—amplitude, frequency, and damping—and solutions) are saved; other program parameters, such as scaling and sound parameters, are not saved.

When you are saving a base simulation or a time series for the first time or are using the **Save As** command, you can use the Delimiter pop-up menu to change the delimiter Bouncing Ball uses to save the file, that is, the character(s) that separate the pieces of data for a given solution or sampling (line) in the text file. Initially Bouncing Ball uses spaces, but it can also use tabs, commas, or carriage returns. The choice of delimiter depends on the application(s) you plan to export the data into. For instance, to export the data to a spreadsheet, tabs or commas are usually the best choice. If you do not plan to export the data to any other application, spaces are probably best. You can set the default delimiter with the **Save Defaults** command.

Bouncing Ball saves special simulations (bifurcation diagrams, basins of attraction) in non-text format. It saves all the parameters involved in the simulation, including the current state thereof, so that, if not completed, it can be continued during another session.

**Save As**

The **Save As** command saves new simulations or new versions of existing ones. After choosing **Save As**, the standard **Save As** dialog box will appear. If the simulation already has a name, Bouncing Ball proposes it in the text box. To accept the proposed name, click the Save button. To save the simulation under a different name, edit the proposed name or type a new one. Click the Drive and/or Eject buttons to save the simulation on another disk.

Refer to the **Save** command for a discussion of how Bouncing Ball saves files, including a discussion of the Delimiter pop-up menu.

**Save Defaults**

The **Save Defaults** command saves the current settings of various Bouncing Ball parameters in a default settings file. It also makes the current window configuration the default configuration. It creates a file entitled “Bouncing Ball Defaults” in the current system folder; when Bouncing Ball starts up, it looks in the system folder for this file and uses the configuration stored therein.

The default settings that Bouncing Ball sets are the following: the initial conditions and table parameters; the iteration parameters, the maximum number of table cycles in a base simulation and the number of decimal places of accuracy used; whether to implement the "graph while opening" option for opening saved Bouncing Ball base simulations; the delimiter used when saving base simulations; if running under MultiFinder, whether to run in the background, and, when running in the background, whether to run quickly or slowly, and whether to use sound; whether to suppress save warnings; whether to use the auto ⌘-key feature; the viewed phase range; whether to use the automatic maximum velocity in graphing, and, if not, what maximum velocity to use; how fast to run the base simulation; the scaling parameters for the *Trajectory* window; whether to scroll the *Trajectory* window; the index offset used in the *Impact Phases* window; whether to graph the *Phase Space* using the "faster" option; whether to generate time series data, and the time series sampling rate; for the special simulations, whether to graph in all windows and whether to use color; the pattern and color settings; and all the **Sound** menu settings.

If you'd like to have more than one "defaults file," you can copy or move the "Bouncing Ball Defaults" file out of the system folder (or even just rename it), create others similarly, then move whichever one you want to use into the system folder.

Note: to restore the original defaults, simply move the defaults file out of the system folder.

**Page Setup**

The **Page Setup** command allows you to select the paper size, paper orientation, and special printing effects for the document. The special printing effects available depend on which printer is currently selected with the Chooser desk accessory.

**Print**

The **Print** command prints the front window.

Exactly what it prints depends on the window. If the window is a text window (*Impact Data* or *Time Series*), it prints all the text (but not text that has been stored elsewhere—see the discussion of the *Impact Data* window in the section "Bouncing Ball Windows"), printing on multiple pages if necessary.

If the front window is a special window (*Bifurcation Diagram* or *Basins of Attraction*), Bouncing Ball prints all applicable information, even if the window is sized such that all information is not visible.

For any other window, Bouncing Ball prints only what is seen in the window as currently sized.

**Quit**

The **Quit** command ends a Bouncing Ball session. It will prompt you to save any unsaved data; hold Option down when you execute the command for a forced quit—the program will not prompt you to save data.

## Edit Menu

With one exception, Bouncing Ball does not allow standard Macintosh editing commands; these non-functional commands will be disabled (grayed) during Bouncing Ball operation. One has no need to paste information into or cut information out of windows. The **Copy** command does work, however. Also, inside a dialog box, **Cut**, **Copy**, and **Paste** will work; they can be called with their Command-key equivalents. All commands are provided in the menu for compatibility with desk accessories.

**Undo**

Provided for desk accessory compatibility only. Disabled when the front window is a Bouncing Ball window.

**Cut**

Provided for desk accessory compatibility only. Disabled when the front window is a Bouncing Ball window.

**Copy**

For a text window (*Impact Data* or *Time Series*), the **Copy** command copies to the clipboard the current selection.

For a graphics window, the **Copy** command copies to the clipboard exactly what is seen in the window, as currently sized.

**Paste**

Provided for desk accessory compatibility only. Disabled when the front window is a Bouncing Ball window.

**Clear**

Provided for desk accessory compatibility only. Disabled when the front window is a Bouncing Ball window.

**Suppress Save Warnings**

By choosing the **Suppress Save Warnings** command, Bouncing Ball will not warn you when data is about to be lost due to quitting, erasing simulations, or creating new simulations. This command toggles "save-warning suppression"; choosing the command a second time will re-enable save warnings. The current setting of this option can be saved as the default setting with the **Save Defaults** command.

**Auto ⌘-key**

After choosing the **Auto ⌘-key** command, you can issue menu commands by typing their Command-key equivalents *without holding the Command key down*. For instance, you could issue the **Open** command by typing just "o"; you would not need to type "Command-o." This command toggles this feature; after choosing the command a second time, you will have to hold down the Command key to issue menu commands. The current setting of this option can be saved as the default setting with the **Save Defaults** command.

**Background Operation**

The **Background Operation** command allows you to tell Bouncing Ball whether to run in the background under MultiFinder. The command will only appear if you are running under MultiFinder. Initially Bouncing Ball does not run in the background, because some of its background processing interferes too much with the foreground application to be considered truly MultiFinder friendly. However, especially given the amount of time it takes to create the special diagrams, background processing can be a real asset.

The **Background Operation** dialog box allows you to specify whether you want Bouncing Ball to run in the background, and whether you want it to run slowly, minimizing its influence on the foreground application, or to run quickly, maximizing Bouncing Ball's processing speed. You also have the option of suppressing sound when Bouncing Ball is running in the background.

Running Bouncing Ball in the background is especially useful for the very time-consuming generation of basins of attraction and bifurcation diagrams.

The background options set here can be stored as the default settings with the **Save Defaults** command.

For more information on running Bouncing Ball in the background, refer to the "MultiFinder and Memory Considerations" part of the section "Tips and Techniques."

## Control Menu

**Begin Iteration**

The **Begin Iteration** command starts a base simulation.

**Stop Iteration**
**Continue Iteration**

The **Stop Iteration** command stops a base simulation. After being stopped, the **Continue Iteration** will continue it. In some circumstances the simulation cannot be continued. This happens most frequently when the ball is stuck forever to the table and the simulation stops automatically.

**Do One Impact**

The **Do One Impact** command starts or continues a base simulation, whichever is appropriate, finds and graphs one impact, and stops the simulation.

**Play Back Solutions**

The **Play Back Solutions** command plays back the current simulation from the beginning. Because the collisions (roots of the equations of motion) do not have to be calculated again, playing back solutions is noticeably faster than generating them. It also tends to generate smoother graphics, especially in the *Animation* window.

**Initial Conditions**

The **Initial Conditions** command allows you to enter new initial conditions for the simulation. Check the (Re)Start Iteration box if you wish the iteration to begin after you click the OK button. Check the Clear Impacts box if you wish to clear the points plotted so far in the *Impact Map* and *Impact Phases* windows; doing this will in effect issue the **Clear Impacts** command from the **View** menu. The current initial conditions can be made the default ones with the **Save Defaults** command.

**Table Parameters**

The **Table Parameters** command allows you to change the table parameters—amplitude, frequency, and damping—of the simulation. Check the (Re)Start Iteration box if you wish the iteration to begin after clicking the OK button. Changing the table parameters will eliminate all data previously generated. The current table parameters can be made the default ones with the **Save Defaults** command.

**Get Initial from Last**

The **Get Initial from Last** command makes the outgoing ball velocity and phase of the last collision between the ball and the table the new initial conditions, bringing up the **Initial Conditions** dialog box.

**Iteration**

The **Iteration** command allows you to set the decimal places of accuracy used in the iteration, and the maximum number of cycles. The decimal places of accuracy controls the accuracy used to find the roots of the system, the points of collision—the fewer the decimal places, the less accurate but faster the iteration. It also controls how quickly the program decides that the ball is sticking to the table; the higher the accuracy, the longer it takes to decide that the ball is sticking. The maximum number of cycles determines the maximum number of table oscillations that may take place before the iteration ends. The current iteration parameters can be made the default ones with the **Save Defaults** command.

When creating bifurcation diagrams or basins of attraction diagrams, Bouncing Ball insists on at least ten decimal places of accuracy, overriding the current accuracy if necessary. The routines that determine when a periodic solution has been reached require such a level of accuracy.

Note that, because the Bouncing Ball system displays sensitive dependence on initial conditions, different accuracies can lead the same initial conditions to qualitatively different solutions.

## View Menu

**Phase Range**

When you first start Bouncing Ball, it graphs phases (in the *Impact Map* and *Impact Phases* windows) from 0 to 1. While natural, this range is somewhat arbitrary. Often it is helpful to look at phases in another range, say, –0.5 to 0.5. The **Phase Range** command allows you to display a phase range of anywhere from –1 to 0, to 0 to 1, in increments of 0.01. Two buttons, entitled "–.5 – .5" and "0 – 1", allow instant selection of these common ranges. The **Save Defaults** command will save the current phase range as the default one.

**Max. Velocity**

The **Max. Velocity** command allows you to specify the maximum velocity graphed, in the *Impact Map* and *Phase Space* windows. Click Auto to use the default value, which is described in the discussion of the *Impact Map* window in the section "Bouncing Ball Windows." The setting set here can be made the default settings with the **Save Defaults** command.

**Simulation Speed**

The **Simulation Speed** command allows you to slow down the simulation so that the graphing in the *Trajectory*, *Animation*, and *Phase Space* windows can be watched more easily. The dialog box this command brings up gives you a scroll bar on which you can set the speed, from slow to fast. The slowest speed corresponds to approximately one table period every ten seconds. If the fastest speed is selected, the speed is limited only by the speed of the computer. The **Save Defaults** command will remember the last speed selected as the default speed.

**Trajectory Scaling**

The **Trajectory Scaling** command allows you to set the "number of table periods in full screen" and the "number of table amplitudes in full screen" in the *Trajectory* window. The number of table periods in full screen represents the number of periods that would be seen horizontally in the *Trajectory* window in its largest (zoomed out or "full screen") position. The number of table amplitudes in full screen represents the number of table amplitudes that would be seen in the *Trajectory* window in "full screen position." The latter scaling is also used in the *Animation* and *Phase Space* windows, as well as to determine a lower limit on the default maximum velocity shown in the *Impact Map* window. The **Save Defaults** command saves the current trajectory scaling parameters as the default ones.

**Scroll Trajectory**

Under the default settings, the graph in the *Trajectory* window clears itself and starts over when the traces reach the right edge. With the **Scroll Trajectory** command, you can tell Bouncing Ball to scroll the *Trajectory* window when it reaches the edge. This is a toggle command; successive executions of the command turn the scrolling off and on. While the scrolling has a nice effect, it tends to slow down graphing in that window by approximately a factor of four, even when the *Trajectory* window is closed, which can slow down the whole simulation by up to a factor of two, depending on which windows are open. The **Save Defaults** command makes the current value of this option the default value.

**Clear Impacts**

The **Clear Impacts** command clears the data in the *Impact Map* and *Impact Phases* windows, so that only subsequent impacts are displayed there. Use this command if the windows become too cluttered. To redisplay the impacts that were calculated prior to this command, choose **Show All Impacts** from this menu.

**Show All Impacts**

The **Show All Impacts** command undoes the effect of the **Clear Impacts** command above; all solutions that had been generated since the beginning of the simulation will be displayed in the *Impact Map* and *Impact Phases* windows.

**Impact Phases Offset**

The **Impact Phases Offset** command allows you to enter an integer that determines which previous impact phase the current impact phase is graphed against in the *Impact Phases* window; refer to the discussion of the window in the section “Bouncing Ball Windows.” The current setting of this variable can be made the default setting with the **Save Defaults** command.

**Impact Map Magnification**

The **Impact Map Magnification** command allows you to view, in the *Impact Map* window, only a portion of the entire impact map, gaining greater detail. You specify minimum and maximum values for both phase and velocity. Press any of the Auto boxes for a corresponding default value.

Click the NO MAGNIFICATION button to use the normal (unmagnified) view.

A convenient way to call up this command is by dragging around the area to be magnified in the *Impact Map* window. Doing this will bring up the **Impact Map Magnification** dialog box with appropriate endpoints.

**Impact Phases Magnification**

The **Impact Phases Magnification** command allows you to view, in the *Impact Phases* window, only a portion of the entire impact phases chart, gaining greater detail. You specify minimum and maximum values for each phase axis. Press any of the Auto boxes for a corresponding default value.

Click the NO MAGNIFICATION button to use the normal (unmagnified) view.

A convenient way to call up this command is by dragging around the area to be magnified in the *Impact Phases* window. Doing this will bring up the **Impact Phases Magnification** dialog box with appropriate endpoints.

**Faster Phase Space**

The **Faster Phase Space** command noticeably speeds up graphing in the *Phase Space* window, at the expense of lessened accuracy. The **Save Defaults** command makes the current setting of this option the default one.

**Clear Phase Space**

The **Clear Phase Space** command clears the data in the *Phase Space* window. The cleared data cannot be restored (though it could be recreated).

**Use Time Series**

The **Use Time Series** command allows you to turn on and off the generation of time series data. If the *Time Series* window is not open and you ask to turn on the data, you will be asked if you want to open the window. The **Save Defaults** command makes the current setting of this option the default one.

**Time Series**

With the **Time Series** command, you can set the sampling rate of the time series data, in sampling times per period. Click the Generate Times Series Data box to start generating time series data. If you request that data be generated, and the *Time Series* window is closed, you will be asked if you want to open the window. The current sampling rate can be made the default one with the **Save Defaults** command.

**Clear Time Series**

The **Clear Time Series** command clears the data in the *Time Series* window. The data cannot be restored (though it could be recreated).

## Special Menu

**Start Bifurcation Diagram**

The **Start Bifurcation Diagram** command begins the creation of the current bifurcation diagram. If the *Bifurcation Diagram* window is not open, you will be asked if you want to open the window.

**Stop Bifurcation Diagram**
**Continue Bifurcation Diagram**

While a bifurcation diagram is running, the **Stop Bifurcation Diagram** command stops it. **Continue Bifurcation Diagram** continues a bifurcation diagram that has been stopped before completion. If you issue the **Continue Bifurcation Diagram** command and the *Bifurcation Diagram* window is not open, you will be asked if you want to open the window.

**Bifurcation Diagram**

The **Bifurcation Diagram** command brings up the dialog box that allows you to set all the parameters of a bifurcation diagram. Please refer to the section "Types of Simulation" for a discussion of this dialog box. If you use this command to start a bifurcation diagram and the *Bifurcation Diagram* window is not open, you will be asked if you want to open the window.

**Bif. Diagram Phase Range**

The **Bif. Diagram Phase Range** command allows you to set the phase range displayed in an existing bifurcation diagram. This command is only active if there is a current bifurcation diagram, if phase is being graphed on the vertical axis (not velocity), and if the range of phases shown on the phase axis is the full range (whether from 0 to 1, –0.5 to 0.5, or any other range of extent 1.0).

The dialog box this command brings up works the same as the dialog box of the **Phase Range** command.

So that the graphs are drawn accurately, the phase range chosen is rounded so that it falls on an exact pixel boundary; thus a specified range of –0.31 to 0.69 might create an actual range of –0.3072884... to 0.6927116.... The latter is the range that would be shown in a subsequent call to the **Bifurcation Diagram** command.

**Clear Bifurcation Diagram**

The **Clear Bifurcation Diagram** command clears the *Bifurcation Diagram* window and resets all bifurcation diagram parameters. Because this command is destructive, you are given the option of saving the current bifurcation diagram first.

**Graph in All Windows**

The **Graph in All Windows** command instructs Bouncing Ball to toggle whether it displays the progress of the bifurcation diagram in all the windows. In constructing the bifurcation diagram, Bouncing Ball actually runs many base simulations; one can watch these simulations as the bifurcation diagram progresses. On the other hand, often one will not want to watch these simulations, as they significantly increase the time required to complete the bifurcation diagram. Use the **Save Defaults** command to make the current setting of this option the default setting.

**Use Color**

The **Use Color** command toggles whether the current bifurcation diagram is displayed in color. This menu item will be disabled (grayed) if the current monitor is not a color one with at least 4-bit color (16 colors) enabled. *Note*: If the monitor is set to display gray scale, at 4 bits or more, this option will be enabled, and, instead of colors, the bifurcation diagram will be displayed in shades of gray. Use the **Save Defaults** command to make the current setting of this option the default setting.

**Start Basins of Attraction**

The **Start Basins of Attraction** command begins the creation of the current basins of attraction diagram. If the *Basins of Attraction* window is not open, you will be asked if you want to open the window.

**Stop Basins of Attraction**
**Continue Basins of Attraction**

While a basins of attraction diagram is running, the **Stop Basins of Attraction** command stops it. **Continue Basins of Attraction** continues a basins of attraction diagram that has been stopped before completion. If you issue the **Continue Basins of Attraction** command and the *Basins of Attraction* window is not open, you will be asked if you want to open the window.

**Basins of Attraction**

The **Basins of Attraction** command brings up the dialog box that allows you to set all the parameters of a basins of attraction diagram. Please refer to the section "Types of Simulation" for a discussion of this dialog box. If you use this command to start a basins of attraction diagram and the *Basins of Attraction* window is not open, you will be asked if you want to open the window.

**Basins of Attr. Phase Range**

The **Basins of Attr. Phase Range** command allows you to set the phase range displayed in an existing basins of attraction diagram. This command is only active if there is a current basins of attraction and if the range of phases shown on the phase axis is the full range (whether from 0 to 1, –0.5 to 0.5, or any other range of extent 1.0).

The dialog box this command brings up works the same as the dialog box of the **Phase Range** command.

So that the graphs are drawn accurately, the phase range chosen is rounded so that it falls on an exact step size boundary; thus a specified range of –0.31 to 0.69 might create an actual range of –0.3072884... to 0.6927116.... The latter is the range that would be shown in a subsequent call to the **Basins of Attraction** command.

**Clear Basins of Attraction**

The **Clear Basins of Attraction** command clears the *Basins of Attraction* window and resets all basins of attraction parameters. Because this command is destructive, you are given the option of saving the current basins of attraction diagram first.

**Graph in All Windows**

The **Graph in All Windows** command instructs Bouncing Ball to toggle whether it displays the progress of the basins of attraction diagram in all the windows. In constructing the basins of attraction diagram, Bouncing Ball actually runs many base simulations; one can watch these simulations as the basins of attraction diagram progresses. On the other hand, often one will not want to watch these simulations, as they significantly increase the time required to complete the basins of attraction diagram. Use the **Save Defaults** command to make the current setting of this option the default setting.

**Use Color**

The **Use Color** command toggles whether the current basins of attraction diagram is displayed in color or with patterns. This menu item will be disabled (grayed) if the current monitor is not a color one with at least 4-bit color (16 colors) enabled. *Note*: If the monitor is set to display gray scale, at 4 bits or more, this option will be enabled, and, instead of colors, the basins of attraction will be displayed in shades of gray. Use the **Save Defaults** command to make the current setting of this option the default setting.

**Patterns**

The **Patterns** command allows you to specify the gray patterns used to represent the different types of solutions in the basins of attraction diagram, when color is off. Use the **Save Defaults** command to save the current pattern-solution relationship as the default one.

**Colors**

The **Colors** command allows you to specify the colors used to represent the different types of solutions in the basins of attraction diagram and the bifurcation diagram, when color is on. Use the **Save Defaults** command to save the current color-solution relationship as the default one.

**Print Pattern List**

The **Print Pattern List** command prints a legend of the patterns representing the different types of solutions.

## Sound Menu

There are actually two **Sound** menus, one containing "simple options," the other "advanced options." The simple options only allow you to turn sounds off and on, while the advanced options allow you some control over the pitches of the sounds.

NOTE: To turn off all sound, eliminate the checks next to the **Phase Tones** command and the **Velocity Tones** command, if any, by choosing the appropriate command.

The **Save Defaults** command will remember all the current settings of this menu as the default settings.

**Phase Tones**
**Velocity Tones**

The **Phase Tones** and **Velocity Tones** commands toggle Bouncing Ball's sound. With "phase tones" on, Bouncing Ball creates, for each impact, a sound corresponding to the phase of that impact. With "velocity tones" on, Bouncing Ball creates a sound corresponding to the ball's velocity after impact, in relation to the minimum and maximum velocity values being used, as shown in the *Impact Map* window (unmagnified). See the other commands from this menu for descriptions of the sounds created. Initially, phase tones are on, velocity tones off.

**Advanced Options**

The **Advanced Options** command adds the following commands to the **Sound** menu.

**One Octave**

[Using advanced options] With the **One Octave** command, the frequency of the sound(s) Bouncing Ball creates (for each collision) ranges from 256 Hz to 512 Hz as the phase of the collision ranges from the low end of the phase range (as set by the **Phase Range** command) to the high end, or as the ball velocity after impact ranges from the minimum to the maximum velocity shown in the *Impact Map* window. See the **Phase Tones** and **Velocity Tones** commands above to turn sound on and off; see below for a description of how the sound frequencies are calculated.

**Three Octaves**

[Using advanced options] With the **Three Octaves** command, Bouncing Ball first calculates the frequency that would be generated using the **One Octave** command above. Then, for phase tones only, Bouncing Ball lowers that frequency an octave if the successive collisions are less than one period apart and raises it an octave if the successive collisions are more than two periods apart. The total range of tones is, therefore, three octaves.

**Major**
**Minor**
**Chromatic**
**Continuous**

[Using advanced options] These four commands determine the exact frequency of sound created by Bouncing Ball. The **Major** command divides the appropriate range (phase from 0 to 1, velocity from the minimum to the maximum velocity, as shown in the *Impact Map* window) into seven equal areas, corresponding to the seven tones of the major scale. The **Minor** command acts similarly, dividing the range into areas corresponding to the seven tones of the minor scale. The **Chromatic** command divides the range into twelve areas, corresponding to the twelve tones of the chromatic scale. With the **Continuous** command, Bouncing Ball creates a sound that ranges logarithmically from 256 to 512 Hz as the phase ranges from 0 to 1 or as the velocity ranges from the minimum to the maximum velocity.

**Simple Options**

The **Simple Options** command removes from this menu all commands except the first two, and adds the **Advanced Options** command, which can in turn be used to restore the removed commands.

## Windows Menu

After the first three commands, which are described below, the **Windows** menu lists all of Bouncing Ball's windows, placing a diamond next to the windows that are open. Choosing one of these menu items brings the named window to the front, making it visible if necessary.

**Close Window**
**Close All Windows**

The **Close Window** command closes the front window. If the Option key is depressed, the command becomes **Close All Windows** and will close all open windows (holding down Option while clicking in the front window's close box will do the same thing). If there are no open windows, this command is grayed.

**Arrange Windows**

The **Arrange Windows** command arranges the currently-open windows in a standard format. If there are no open Bouncing Ball windows, this command is grayed.

**Default Configuration**

The **Default Configuration** command puts Bouncing Ball's windows in the configuration stored in the "Bouncing Ball Defaults" file in the current system folder (see **Save Defaults** command). If there is no such file, it sizes the *Trajectory*, *Impact Data*, *Impact Map*, and *Animation* windows equally, and puts one into each quarter of the screen.

# TIPS AND TECHNIQUES

This section contains a series of tips and techniques to make your use of Bouncing Ball easier and more enjoyable.

## Speeding Up the Simulation

Closing any or all of the following windows will speed up the simulation, with the windows listed first helping the most: *Time Series* (or just turning off the time series generation), *Phase Space*, *Animation*, and *Trajectory*. If you wish to keep the *Phase Space* window open, turning on the "Faster Phase Space" option (**Faster Phase Space** command, **View** menu) will help. Other techniques to speed up the simulation include turning the sound off and turning off trajectory scrolling (which initially is off—see the **Scroll Trajectory** command, **View** menu). Of course, if the simulation has been slowed with the **Simulation Speed** command (**View** menu), you can speed it up with that command. While opening base simulation files, leave the Graph while opening box unchecked; if it is checked, Bouncing Ball will maintain all windows as it opens, which can slow the operation down tremendously.

Bouncing Ball's special simulations, being composed as they are of many base simulations, tend to take a lot of time under any circumstances, especially with a slow Macintosh. Six hours is not at all uncommon for a bifurcation diagram, even more time for a basins of attraction diagram. To generate special simulations as fast as possible, do not select the appropriate "graph in all windows" command and turn the sound off. In addition, with basins of attraction diagrams, using a larger step size will speed it up tremendously, albeit with diminished resolution. Using the "Faster" option will also speed it up tremendously, especially if you include the actual attractors in the basins of attraction graph range. Finally, if you are using a color system, creating basins of attraction in color is a little slower than in black and white (patterns); you can create them in black and white and later display them in color.

## MultiFinder and Memory Considerations

Bouncing Ball can run in the background under MultiFinder, but only if you explicitly instruct it to, with the **Background Operation** command from the **Edit** menu. See the description of that command for more information.

There are several considerations that should be given to memory usage with Bouncing Ball under MultiFinder. First of all, if you are not familiar with memory partitioning under MultiFinder, it is set with the Get Info command in the Finder; please refer to your MultiFinder manual for more information. Bouncing Ball's default memory partition is 900K. That should be sufficient to run all three types of simulations, and to create base simulations of a couple thousand iterations or more.

Of that 900K, about 250K is used to create basins of attraction diagrams. While generating these diagrams, Bouncing Ball maintains many off-screen bitmaps, which consume large amounts of memory. If you do not create basins of attraction diagrams, you can safely run Bouncing Ball with 600K; as little as 512K would, in fact, probably be sufficient.

If you are using a larger monitor or multiple monitors, you should increase the default partition size by the following amount: Multiply the number of pixels the larger monitor displays horizontally by the number of pixels it displays vertically to arrive at its resolution, its total number of pixels. In the case of multiple monitors, use the total numbers of pixels that would be displayed by a large monitor just barely encompassing all your monitors (as arranged with the Monitors cdev in the Control Panel). Subtract 200,000 from this resolution and multiply the resulting number by 0.002 (0.001 if you will not create any basins of attraction diagrams). The result of this calculation represents the additional memory (in "K") that should be added to Bouncing Ball's memory partition. The calculation is reproduced below:

Additional Memory = (Resolution – 200,000) * 0.002 (0.001 if no basins of attraction).

In addition, if you wish to create large base simulations (> 2000 iterations), you should increase the default partition size by 80K for each 1000 iterations.

Of course, depending on the memory available in your computer, the above calculations may imply certain restrictions on the operation of Bouncing Ball; for example, creating a basins of attraction diagram at full window size on a Macintosh with a large monitor but only 1 MB of memory, running without MultiFinder, should not be attempted.

One other consideration: If you open a bifurcation diagram created and sized in a window larger than your current monitor, Bouncing Ball may create "memory islands," which can cause it to be unable to fully use the available memory. Beware of this if you create special files on large monitors, then open them on small monitors. By the way, the enlarged memory values given above would partially apply to such situations. Thus, even though you may be using a small Macintosh monitor, you will require much of the memory of a monitor large enough to display the entire large-sized special file.

## Bouncing Ball and Other Programs

Bouncing Ball can export data in several ways. Graphics and text can both be exported using the **Copy** command and the Clipboard. Files saved from Bouncing Ball's *Impact Data* or *Time Series* windows, being text files, can be easily shared with many programs. The **Save As** command allows you to specify the data delimiter, to ease data transfer. Bifurcation diagram and basins of attraction files, on the other hand, use a proprietary format. The graphics in these files are stored as PICT resources; an ambitious user could use a resource editor to access these graphics and transfer them to another application, perhaps to gain greater color control, or to make color separations.

For the most part, it is not possible to import data into Bouncing Ball. Technically, Bouncing Ball can try to open any text file, and if that file contains data in Bouncing Ball format, it will open it. If the data is not actual Bouncing Ball data, however, this is likely to result in erratic performance.

Bouncing Ball appears to coexist peacefully with all popular INITs, cdevs, and desk accessories. If you begin a simulation and leave the computer alone, and if you use a screen saver, the screen saver will blank the screen while Bouncing Ball is creating the simulation. Bouncing Ball continues to create data after the screen saver is activated. When the screen saver is deactivated, Bouncing Ball will properly display all the data.

## Miscellaneous

Warning or confirmation dialog boxes are generated by certain commands. For example, several time series commands and special commands trigger dialog boxes. For many of these commands, holding the Shift key down when executing the command will bypass the appropriate dialog box, automatically making the default response. For some of these, holding the Option key down when executing the command will effectively answer "No" to the dialog box. For instance, if you hold Shift down while executing the **Quit** command, the program will automatically save all current data before quitting, while if you hold Option down, the program will quit while neither prompting you to save data nor saving data. Shift has priority over Option; if both are held down, the Option key will be ignored. Either Shift or both Shift and Option work in this fashion with the following commands: **New**, **Open**, **Quit**, **Table Parameters**, **Get Initial from Last**, **Clear Bifurcation Diagram**, **Clear Basins of Attraction**, both **Graph in All Windows** commands, commands that start or continue basins of attraction or bifurcation diagrams, and several time series-related commands. Shift also works when magnifying in the *Impact Map*, *Impact Phases*, *Bifurcation Diagram*, and *Basins of Attraction* windows (to bypass the corresponding dialog box), and when clicking initial conditions in the *Impact Map* window.

In any dialog box, pressing Command-letter will have the effect of clicking the "first" control (button, radio button, check box) whose title begins with that letter. The order of these controls is determined internally; in general, the OK (default) and Cancel (or analogous) buttons come first, followed by the others in order from top to bottom, left to right. In a dialog box with no text input (edit text) fields, pressing a letter without Command will click the first radio button or check box whose title begins with that letter. Command-period or Escape will cancel dialog boxes. Tab takes you forward through the text input fields; Shift-Tab takes you backward through these fields.

Command-period (□-.) will stop the current simulation, whether a base simulation, a bifurcation diagram, or a basins of attraction diagram. It will also stop the file-opening of a base simulation.

In the **Phase Range** dialog boxes, pressing the Left and Right Arrow keys decrements/increments the phase range by 0.01; pressing the Up and Down keys decrements/increments by 0.1. Similarly, in the **Simulation Speed** dialog box, pressing the Left and Right Arrow keys moves the scroll bar's "thumb" by a small amount in the corresponding direction; pressing the Up and Down keys moves the thumb by a larger amount.

*For Advanced Users:* If you wish to change the patterns used to display the ball, the table, and the table's frame in the *Animation* window (default: black and 50% gray, respectively), you can use a resource editor to modify the first three patterns, respectively, in 'PAT#' ID 132 (entitled "Ball, Table").

## Program Problems

Perhaps Bouncing Ball's biggest problem is that it only handles mouse clicks and other input at impact time. For instance, while a single trajectory is being shown in the *Animation* window, mouse clicks will not be acted upon. They are stored up and are handled at the time the impact is made. This problem is accentuated when the ball bounces high off the table.

Bifurcation diagrams and basins of attraction diagrams may not print properly on an Apple LaserWriter SC™ with some versions of the printer driver. If you use the latest version of the printer driver, they should print fine.

While using color, in particular on a basins of attraction diagram, the Macintosh's cursor may disappear for a few seconds. Don't be alarmed—it will come back.

If you find any bugs or other program problems, please notify the publisher.

# SELECTED ERROR MESSAGES

Most of Bouncing Ball's error messages should be self-explanatory. Below are a few error messages that may require additional explanations and/or suggestions.

**Could not load solution patterns. Will use smaller set.**

Bouncing Ball was unable to read the patterns it uses in the basins of attraction diagram from its own resource fork ('PAT#' ID 131, "Eight Grays"). It will substitute the standard 100% (black), 75%, 50%, 25%, and 0% (white) grays the Macintosh automatically provides (note that this procedure yields only five different grays, less than the usual eight; however, in general this will be enough to give each type of solution found a different pattern). If you are so inclined, you could use a resource editor to inspect that resource; there should be eight progressively lighter patterns in it.

**Initial velocity less than table velocity**

When starting an iteration, the ball must start out going as fast as or faster than the table; otherwise, the ball would sink *through* the table. You may get this error message at times when, analytically, the ball and the table should be traveling at exactly the same speed. If this happens, just increase the initial ball velocity by a small amount.

**Not a Bouncing Ball file**

The text file Bouncing Ball is trying to open does not have a valid Bouncing Ball header (which contains the amplitude, frequency, and damping coefficient). This could be because the file has nothing to do with Bouncing Ball, since Bouncing Ball's **Open** dialog includes all text files—refer to the discussion of the **Open** command. If the file should be a proper Bouncing Ball file, you could use a word processor to look at the file, compare it to a file that Bouncing Ball can open, make any required changes, save it as a text file, then open it inside Bouncing Ball again.

**Root finding error; Bouncing Ball will assume sticking solution**
**Root finding error: no zero on interval**

Either of these messages means that Bouncing Ball was unable to find a solution for the current iteration. Ideally these messages will never come up; if they do, it is likely due to a problem elsewhere in the program. The first message appears if you are running a bifurcation diagram or a basins of attraction diagram; in this case, the program assumes it has found a sticking solution (this error is most likely to occur in a sticking region, where the bounces are small) and proceeds just as if it had found a normal sticking solution. The second message appears while running a base iteration; in this situation the iteration will stop and may not be continued (though it could be restarted).

**Sorry—not enough memory to run Bouncing Ball**

If you are running under MultiFinder, this command means that Bouncing Ball's memory partition is too small. If not, the Macintosh being used does not have sufficient memory to run Bouncing Ball. Refer to the discussion of memory usage in the section "Tips and Techniques."

**Unable to open entire file**

One of the solutions in the file Bouncing Ball is opening is not in valid format. Each solution should consist of the hit number (an integer) and three floating point numbers, the phase, the velocity, and the time. To correct this problem, inspect the file with a word processor, looking where Bouncing Ball stopped loading the data, and try to find an invalid number, or an incomplete line, or something of that nature. If you are able to find and correct the problem, save it as a text file and re-open it in Bouncing Ball.

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