Quadratic Map
User's Guide

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INTRODUCTION

Quadratic Map is a program written for the Apple® Macintosh™ computer that iterates the quadratic map

$$x_{n+1} = \lambda x_n(1 - x_n).$$

Quadratic Map 1.0 requires the use of a Macintosh with at least 512 kilobytes (512K) of memory, at least the 128K version of ROM (all Macintoshes except the original 128K Macintosh and the original Macintosh 512 have such a version), and the 6.0 System set (System 6.0 and Finder 6.1) or later.

This manual explains how to use Quadratic Map. Before you start, you should already know how to perform basic operations with your Macintosh. You should know how to use the Finder™ and the mouse, manipulate windows, scroll, pull down menus, and choose commands. If you need information about any of these topics, consult your owner's guide.

This manual has six main sections:

- Getting Started
- Types of Iteration
- Quadratic Map Windows
- Quadratic Map Menus
- Tips and Techniques
- Selected Error Messages

Getting Started shows you how to create iterations and describes the sample files that accompany Quadratic Map.

Types of Iteration describes the two types of iterations Quadratic Map can run.

Quadratic Map Windows describes the windows Quadratic Map uses to display data from the iterations.

Quadratic Map Menus contains a directory of all Quadratic Map menu commands, arranged in the order they appear on the pull-down menus.

Tips and Techniques describes various ways to tailor your use of Quadratic Map.

Selected Error Messages lists the error messages that Quadratic Map generates and describes what to do when they appear.

GETTING STARTED

1. Start the Macintosh by turning it on. If you do not have a hard disk, insert a Macintosh system disk.
2. Insert the Quadratic Map disk and open it, if it is not already open.
3. Open the Quadratic Map folder.
4. Open the Quadratic Map application.
5. After the program starts up, choose **Initial Condition** from the **Control** menu. Change the initial condition to some other value, or leave it the same, then click the OK button.

The iteration should now be running. Note that there are four windows currently displayed. The upper left window, *Phase Space*, contains a plot of the *n*+1st value versus the *n*th value. The upper right window, *Untitled* (when you open an iteration or save one, this window takes on the name of the file; it will often be called the *Iteration Data* window), lists the system parameter—lambda—and, for each iteration, the sequential number of the iteration and the iteration value. The lower left window, *Embedded Phase Space*, initially contains a plot of the *n*+2nd value versus the *n*th value. The lower right window, *Orbit*, plots, for each iteration, the iteration value versus the sequential number of the iteration.

In addition to the initial value, you can control the system parameter lambda by choosing **System Parameter** from the **Control** menu.

For instance, to illustrate the period-doubling route to chaos in the quadratic map system, we increase lambda in successive iterations. First reset the initial condition to the original value, 0.5. The initial value for lambda, which leads to a period one orbit, is 3. To generate a period two orbit, increase lambda to 3.2 by choosing **System Parameter** from the **Control** menu and changing lambda from 3 to 3.2. Now click OK to run the iteration with this new parameter value. Follow the same procedure to increase lambda to 3.9 to produce a chaotic trajectory.

An alternate way to choose a new initial condition is to click in the body of the phase space; the **Initial Condition** dialog box will come up with an initial value corresponding to the point of the mouse-click (as long as the click doesn't hit the 45° line $x_{n+1} = x_n$ or the parabola that forms the basis for the quadratic map). In addition, clicking in certain areas of the various windows calls forth appropriate dialog boxes. These active areas are known as "hot spots." Such hot spots are located along the two axes of the *Phase Space* window. The section "Quadratic Map Windows" discusses these areas further.

Now we'll look at Quadratic Map's other type of iteration, which is based on the iteration described above.

Bifurcation Diagram

To create a *bifurcation diagram*, follow these steps:

1. Use the **Initial Condition** command to restore the original initial value, 0.5.
2. Use the **System Parameter** command to change the value of the system parameter lambda to 2.
3. Clear the current data by choosing **New** from the **File** menu.
4. Open the *Bifurcation Diagram* window by choosing **Bifurcation Diagram** from the **Windows** menu.
5. Choose **Bifurcation Diagram** from the **Special** menu and click OK.

The program will generate a diagram showing how the solutions to the Quadratic Map system vary as the system parameter lambda varies. The bifurcation diagram stops automatically after it has moved through the specified lambda range.

Sample Files

To help you quickly see some of its capabilities, Quadratic Map is packaged with the following sample files. You can either double-click on these sample files or open them with the **Open** command in the **File** menu.

Period 2	An iteration that converges to a period two solution; $\lambda = 3.2$
Period 16 (lengthy)	An iteration that converges to a period sixteen solution; $\lambda = 3.568$ (it contains 700 iterations, so it takes a while to open, especially on slower Macintoshes)
Strange Attractor (lengthy)	An example of chaos; $\lambda = 4$ (it contains 1000 iterations, so it takes a while to open, especially on slower Macintoshes)
Bifurcation Diagram 0–4	A bifurcation diagram in which lambda is incremented from 0 to 4
Bifurcation Diagram 2.9–4	A bifurcation diagram that essentially magnifies the “period-doubling route to chaos” shown in the first bifurcation diagram file

TYPES OF ITERATION

Quadratic Map can perform two types of iteration. The central iteration to Quadratic Map is the straightforward one described in the beginning of the section "Getting Started," wherein you set the system parameter and the initial condition, then watch the iteration in any combination of four windows. This type of iteration will be called the "base" iteration.

The other type of iteration, sometimes referred to as a "special" iteration, is the bifurcation diagram. It is based on the base iteration and is displayed in its own window.

Base Iteration

The base iteration is a computer simulation of a logistic map iteration. Using the **Initial Condition** and **System Parameter** menu commands, you set the parameters, then let the computer iterate the solutions.

As the iteration progresses, you can watch the data displayed in the *Phase Space*, *Iteration Data* ("*Untitled*" at the beginning), *Embedded Phase Space*, and *Orbit* windows. The section "Quadratic Map Windows" describes how each of these windows displays the data.

Bifurcation Diagram

In the *Bifurcation Diagram*, you select a *range* of values for the system parameter lambda. The program then starts the Quadratic Map system at the beginning of this range. It lets the base iteration run for a certain number of iterations (to eliminate the "transient" solution), then charts the values of the remaining iterations on the vertical axis versus lambda on the horizontal axis. It then increments lambda (by a value corresponding to one screen pixel) and repeats the process, until it reaches the end of the parameter range.

WARNING: The bifurcation diagram runs fairly slowly, especially on slow Macintoshes.

What one finds in the bifurcation diagram is that, as the parameter is changed, the type of solution the Quadratic Map system reaches can change. For instance, in the lambda range of 2–4, with an initial value of 0.5, the system will go through a period one solution, a period two solution, a period four solution, a period eight solution, and so on until it reaches chaos.

To create a bifurcation diagram, choose **Bifurcation Diagram** from the **Special** menu and complete the dialog box as described below.

1. Specify the range of values for the system parameter lambda.
2. Specify the range of iteration values to be displayed.
3. Specify the initial value for the first iteration.
4. Specify the number of iterations you want the iteration to run before charting the data, for each value of lambda. Checking the Auto box puts in the word "Auto" and gives a value based on lambda—higher lambdas give higher numbers of iterations. In addition, with Auto checked, if after reaching the automatic value Quadratic Map determines that it has not reached periodicity, but that it will probably reach periodicity if continued, it will increase this number of initial iterations. Checking Auto tends to give the most accurate results. *Note*: If Quadratic Map determines that periodicity has been reached before reaching the given number of iterations, it will automatically start charting the data.
5. Specify the number of iterations you want Quadratic Map to chart at each value of lambda. Checking the Auto box puts in the word "Auto." If you choose the automatic option, Quadratic Map charts several iterations, then charts an additional number of iterations based on the dispersion of those first charted iterations. If, because of a periodic solution, Quadratic Map decides it does not need to display any more solutions, it stops. *Note*: In a chaotic system, in full-screen view on a small Macintosh, the automatic value described above can easily lead to 500 or more iterations, which can be quite time consuming. A value of 100 or so should lead to satisfactory results in most circumstances.
6. Select the window size desired: if As-Is is selected, the size of the bifurcation diagram will be such that it will fit into the *Bifurcation Diagram* window as currently sized; if Full Screen is checked, the bifurcation diagram will take up almost the whole screen (it will just fit into a window that has been "zoomed out"). *Note*: The status of these radio buttons will not physically change the size of the *Bifurcation Diagram* window, but it will affect how the scroll bars of that window, if any, operate.
7. Set these miscellaneous bifurcation diagram parameters:
    a. Graph in All Windows—As remarked above, the bifurcation diagram actually creates a series of base iterations. Check this box if you wish to display the base iterations in all appropriate windows, as described in the "Base Iteration" part of this section. This allows you to see in detail what's happening at each value

of the system parameter; however, it greatly slows down the creation of the bifurcation diagram.

b. (Re)Start—Check if you want to start the bifurcation diagram after clicking OK.

c. Use Color—Check if you want the bifurcation diagram to display in color. When displayed in color, each type of solution (refer to the "Types of Solutions" discussion of this section for a description of the different solution types) displays in a different color. If you do not have a color monitor, or if it is not set for 4- or more bit color (16 or more colors), this button will be disabled (grayed). *Note*: If your monitor is set for gray-scale, for 4 or more bits, you can implement "color"—it will simply use different shades of gray.

8. If color is being used, pressing the Colors button will bring up the same dialog box as the **Colors** command from the **Special** menu.

9. Press OK to complete the input, Cancel to cancel it.

Types of Solutions

In a bifurcation diagram, Quadratic Map recognizes the following types of solutions:

Unbounded

Period One

Period Two

Period Three

Period Four

Period Eight

Period Sixteen

Other

"Unbounded" solutions are solutions for which the iteration goes to negative infinity. For positive lambda and an initial condition between 0 and 1, lambda must be greater than 4 for this to occur. "Period One" through "Period Sixteen" solutions are just what their names suggest; periodic solutions of the given periods. "Other" solutions, as you might guess, are those that do not fit into any other category. In practice, they are most often chaotic solutions, but can also be other 2^n-periodic solutions, such as period 32, or still other periodic solutions, such as period five. In addition, slow convergence and insufficient iterations can cause Quadratic Map to think that, say, a period two solution is an "other" solution.

QUADRATIC MAP WINDOWS

This section describes the five windows Quadratic Map uses to display data.

Phase Space

For each iteration, Quadratic Map plots in the phase space the value of the current iteration against the value of the previous one. By default the map displays the entire graph range on both axes, as set by the **Graph Range** command. Either range may be changed (and later restored) by magnifying the phase space as described below and in the description of the **Phase Space Magnification** command.

In addition to the points plotted, the *Phase Space* window can also show the parabola the quadratic map is based on (and on which all solutions in this window lie), the 45° line $x_{n+1} = x_n$, the outline of the rectangle that goes from $x_{n+1} = 0$ to $x_{n+1} = 1$ and from $x_n = 0$ to $x_n = 1$, and vertical and horizontal lines that show the path from one solution to the next, alternatively with arrows showing the direction of iteration. Use the **Phase Space** command (**View** menu) to implement these options.

Finally, the window can also display a number line which represents in one dimension the solutions as they are created. Again use the **Phase Space** command to display and set the width of this number line.

Special Features: Clicking inside the phase space will do one of three things. If the click falls close to the parabola defining the quadratic map, it will toggle the display of the parabola. Otherwise, if the click falls close to the 45° line mentioned above, it will toggle the display of that line. If the click hits neither of the above, it will bring up the **Initial Condition** dialog box with a value corresponding to the point of the click. Dragging inside the phase space brings up the **Phase Space Magnification** dialog box, with values corresponding to the dragging endpoints. Hold the Shift key down to bypass the dialog box.

Iteration Data (“Untitled”)

This text window contains, for each iteration, the sequential number and the value of the iteration. Quadratic Map provides no way to control the precision used to display the numbers in this window.

When a base iteration is running, text is being continuously created, so Quadratic Map does not allow you to scroll through or select text. When the iteration is stopped, however, you may scroll through the text, make selections, and copy information to the Clipboard.

When an iteration is saved or opened, this window takes on the name of the corresponding file.

When approximately 32K of text is created (approximately 1450 solutions), most of the text is eliminated from the window (but still stored and able to be saved), and an appropriate message appears at the beginning of the text. The preceding text may be viewed by playing back the solutions (**Play Back Solutions** from the **Control** menu) or by saving it and viewing the saved (text) file in a word processor or other application.

Embedded Phase Space

For each iteration, Quadratic Map plots in the embedded phase space the value of the current iteration against the value of a previous one. You can tell Quadratic Map which previous iteration to use (how many iterations back—the number is called the *embedding*) with the **Embedded Phase Space** command in the **View** menu; the first time you start Quadratic Map, it is the second preceding iteration (2 iterations back). Note that the latter value gives it the same display as the *Phase Space* window. By default the map displays the entire graph range on both axes, as set by the **Graph Range** command. Either range may be changed (and later restored) by magnifying the embedded phase space, as described below and in the description of the **Embed. Phase Space Magnification** command.

In addition to the points plotted, the *Embedded Phase Space* window can also show the 2^n-degree polynomial that results from composing the quadratic map a number of times equal to the embedding (all points will fall on this polynomial), the 45° line $x_{n+k} = x_n$, where k equals the embedding, and the outline of the rectangle that goes from $x_{n+k} = 0$ to $x_{n+k} = 1$ and from $x_n = 0$ to $x_n = 1$. Use the **Embedded Phase Space** command to implement these options.

Special Features: Clicking inside the embedded phase space will do one of two things. If the click falls close to the 2^n-degree polynomial described above, it will toggle the display of the polynomial. Otherwise, if the click falls close to the 45° line mentioned above, it will toggle the display of that line. Dragging inside the embedded phase space brings up the **Embed. Phase Space Magnification** dialog box, with values corresponding to the dragging endpoints. Hold the Shift key down to bypass the dialog box.

Orbit

The *Orbit* window shows the value of the iteration as a function of the iteration index n. It contains a series of dots, moving from left to right, that represent the values of the iteration. The bottom of the window displays the initial condition.

The scaling in this window depends on the values set with the **Graph Range** command from the **View** menu. With that command, you set the numerical range shown by the vertical axis of this window; it automatically scales to fit this range into the window. In addition, the dots may be made larger, and space can be added horizontally between the dots, with the **Orbit** command.

When the dots reach the right edge of the graph, by default the graph is cleared and the traces continue from the left edge. If you prefer, the dots can instead smoothly scroll to the left as data is added to the right. Use the **Scroll Orbit** command in the **View** menu to toggle this scrolling. (Technically, you can make scrolling the default with the **Save Defaults** command.) Scrolling takes considerably more time.

Special Feature: Clicking inside the graph area brings up the **Orbit** dialog box. NOTE: This is not technically considered a hot spot, since it does not invert color when clicked, but it functions like one, and is listed parenthetically in the Hot Spots part of this section.

Bifurcation Diagram

Quadratic Map uses this window to create and display bifurcation diagrams, as discussed in the section "Types of Iteration." The bottom of the window displays the fixed bifurcation diagram parameters, as set in the **Bifurcation Diagram** dialog box, and the initial condition and the value of lambda for the latest iteration.

When a bifurcation diagram is saved or opened, this window takes on the name of the corresponding file.

Special Features: Clicking inside the body of the bifurcation diagram brings up the **Bifurcation Diagram** dialog box. Dragging inside the bifurcation diagram brings up the same dialog box, with range values corresponding to the dragging endpoints (which effectively allows magnification; however, since it cannot magnify existing data, the data must be regenerated). Hold the Shift key down to bypass the dialog box and create the bifurcation diagram for the magnified area.

Hot Spots

Hot spots are areas in Quadratic Map windows that may be clicked in order to execute specific commands. Below is a summary of the hot spots in each window. (Clicking/dragging in the graphing regions of some windows has other effects; see the "Special Features" parts of the window-by-window discussion above.) As used below, "margin" means that region between the graph area and the corresponding edge of the window (not including scroll bars or scroll bar lines).

Window	Hot spot region	Command
Phase Space	Bottom margin	**Phase Space**
	Left margin (while not magnifying)	**Graph Range**
	Left margin (while magnifying)	**Phase Space Magnification**
Iteration Data	Header area	**System Parameter**
Embedded Phase Space	Bottom margin	**Embedded Phase Space**
	Left margin (while not magnifying)	**Graph Range**
	Left margin (while magnifying)	**Embed. Phase Space Magnification**
Orbit	Bottom margin	**Initial Condition**
	Left margin	**Graph Range**
	(Graph area)	**Orbit**
Bifurcation Diagram	Bottom margin	**Bifurcation Diagram**
	Left margin	**Bif. Diagram Phase Range** (if command is active) or **Bifurcation Diagram**
	Whole window (when blank)	**Bifurcation Diagram**

QUADRATIC MAP MENUS

In addition to the requisite **Apple** menu, Quadratic Map has seven menus. The **File** menu contains the standard commands for file manipulation, printing, and quitting. The **Edit** menu contains the standard Macintosh editing commands (though Quadratic Map only implements the **Copy** command; the rest are supplied for desk accessory support), and a few commands best described as preferences commands. The **Control** menu contains the commands that allow you to control Quadratic Map's iteration: starting/stopping it, setting parameters, and so on. The **View** menu allows you to control how you view the iteration; it contains commands to set scaling, hide old data, and so on. The **Special** menu contains the commands that control Quadratic Map's special type of iteration, bifurcation diagrams. The **Sound** menu allows you to control Quadratic Map's sound. The **Windows** menu has commands to control window size and placement and to open windows.

The remainder of this section describes each of Quadratic Map's menu commands.

File Menu

New

The **New** command stops the current iteration and clears the data from all windows. The **New** command closes the current file(s), if any.

Open

The **Open** command opens an iteration stored on the disk and displays it on the screen.

Since Quadratic Map files (from base iterations) are stored as text files, the **Open** dialog box will display all text files. Opening a text file not in Quadratic Map format will cause an error message and will clear all the windows. See the **Save** command for more information on how Quadratic Map saves iterations.

Command-period will abort the opening of a base iteration file.

Opening Quadratic Map files is somewhat different from standard Macintosh opening. Usually, when one opens a Macintosh file, it appears in *its own* window. Quadratic Map files open into *pre-existing* windows. With the exception of size and position, these windows maintain their attributes when a new file is opened; think of these windows as tools to inspect whatever data is displayed, tools that are not reset for new data.

Save

The **Save** command saves the current iteration on the disk from which it was opened (or to which it was "saved as"). Quadratic Map saves the iteration with the name used the last time you saved it, and replaces the old copy of the iteration on the disk. You can only use the **Save** command with iterations that you have previously saved with the **Save As** command; if the iteration has not previously been saved, this command acts like the **Save As** command.

Quadratic Map decides what to save by looking at the active window. If the active window is the *Bifurcation Diagram* window, and there is a current bifurcation diagram, it saves that data. In all other cases, it saves the base iteration data, the data that appears in the *Iteration Data* window.

Quadratic Map saves base iterations as text files, just as the iteration appears in the *Iteration Data* window. The data can thus be easily passed to other applications for further processing. Quadratic Map also saves appropriate window sizes and positions, in such a way that these will not interfere with the transfer to other applications (this window information is stored in the resource fork of the file). Note that, except for window sizes and positions, only the values appearing in the text window (system parameter lambda and solutions) are saved; other program parameters, such as scaling and sound parameters, are not saved.

When you are saving a base iteration for the first time or are using the **Save As** command, you can use the Delimiter pop-up menu to change the delimiter Quadratic Map uses to save the file, that is, the character(s) that separate the pieces of data for a given solution (line) in the text file. When you first start Quadratic Map, it uses spaces, but it can also use tabs, commas, or carriage returns. The choice of delimiter depends on the application(s) you plan to export the data into. For instance, to export the data to a spreadsheet, tabs or commas are usually the best choice. If you do not plan to export the data to any other application, spaces are probably best. You can set the default delimiter with the **Save Defaults** command.

Quadratic Map saves bifurcation diagram files in non-text format. It saves all the parameters involved in the iteration, including the current state thereof, so that, if not completed, it can be continued during another session.

Save As

The **Save As** command saves new iterations or new versions of existing ones. After choosing **Save As**, the standard **Save As** dialog box will appear. If the iteration already has a name, Quadratic Map proposes it in the text box. To accept the proposed name, click the Save button. To save the iteration under a different name, edit the proposed name or type a new one. Click the Drive and/or Eject buttons to save the iteration on another disk.

Refer to the **Save** command for a discussion of how Quadratic Map saves files, including a discussion of the Delimiter pop-up menu.

Save Defaults

The **Save Defaults** command saves the current settings of various Quadratic Map parameters in a default settings file. It also makes the current window configuration the default configuration. It creates a file entitled “Quadratic Map Defaults” in the current system folder; when Quadratic Map starts up, it looks in the system folder for this file and uses the configuration stored therein.

The default settings that Quadratic Map sets are the following: the initial condition and system parameter; the maximum number of iterations in a base iteration; the delimiter used when saving base iterations; if running under MultiFinder, whether to run in the background, and, when running in the background, whether to run quickly or slowly, and whether to use sound; whether to suppress save warnings; whether to use the auto □-key feature; whether to use the automatic graph range, and, if not, what graph range to use; how fast to run the base iteration; all the parameters set in the **Phase Space**, **Embedded Phase Space**, and **Orbit** commands (**View** menu); whether to scroll the graph in the *Orbit* window; for bifurcation diagrams, whether to graph in all windows and whether to use color; the color settings; and all the **Sound** menu settings.

If you'd like to have more than one "defaults file," you can copy or move the "Quadratic Map Defaults" file out of the system folder (or even just rename it), create others similarly, then move whichever one you want to use into the system folder.

Note: to restore the original defaults, simply move the defaults file out of the system folder.

Page Setup

The **Page Setup** command allows you to select the paper size, paper orientation, and special printing effects for the document. The special printing effects available depend on which printer is currently selected with the Chooser desk accessory.

Print

The **Print** command prints the front window.

Exactly what it prints depends on the window. If the window is the text window (*Iteration Data*), it prints all the text (but not text that has been stored elsewhere—see the discussion of the *Iteration Data* window in the section "Quadratic Map Windows"), printing on multiple pages if necessary.

If the front window is the *Bifurcation Diagram* window, Quadratic Map prints all applicable information, even if the window is sized such that all information is not visible.

For any other window, Quadratic Map prints only what is seen in the window as currently sized.

Quit

The **Quit** command ends a Quadratic Map session. It will prompt you to save any unsaved data; hold Option down when you execute the command for a forced quit—the program will not prompt you to save data.

Edit Menu

With one exception, Quadratic Map does not allow standard Macintosh editing commands; these non-functional commands will be disabled (grayed) during Quadratic Map operation. One has no need to paste information into or cut information out of windows. The **Copy** command does work, however. Also, inside a dialog box, **Cut**, **Copy**, and **Paste** will work; they can be called with their Command-key equivalents. All commands are provided in the menu for compatibility with desk accessories.

Undo

Provided for desk accessory compatibility only. Disabled when the front window is a Quadratic Map window.

Cut

Provided for desk accessory compatibility only. Disabled when the front window is a Quadratic Map window.

Copy

For the *Iteration Data* window, the **Copy** command copies to the clipboard the current selection.

For any other window, the **Copy** command copies to the clipboard exactly what is seen in the window, as currently sized.

Paste

Provided for desk accessory compatibility only. Disabled when the front window is a Quadratic Map window.

Clear

Provided for desk accessory compatibility only. Disabled when the front window is a Quadratic Map window.

Suppress Save Warnings

By choosing the **Suppress Save Warnings** command, Quadratic Map will not warn you when data is about to be lost due to quitting, erasing iterations, or creating new iterations. This command toggles "save-warning suppression"; choosing the command a second time will re-enable save warnings. The current setting of this option can be saved as the default setting with the **Save Defaults** command.

Auto □-key

After choosing the **Auto □-key** command, you can issue menu commands by typing their Command-key equivalents *without holding the Command key down*. For instance, you could issue the **Open** command by typing just "o"; you would not need to type "Command-o." This command toggles this feature; after choosing the command a second time, you will have to hold down the Command key to issue menu commands. The current setting of this option can be saved as the default setting with the **Save Defaults** command.

Background Operation

The **Background Operation** command allows you to tell Quadratic Map whether to run in the background under MultiFinder. The command will only appear if you are running under MultiFinder.

The **Background Operation** dialog box allows you to specify whether you want Quadratic Map to run in the background, and whether you want it to run slowly, minimizing its influence on the foreground application, or to run quickly, maximizing Quadratic Map's processing speed. You also have the option of suppressing sound when Quadratic Map is running in the background.

Running Quadratic Map in the background is especially useful for the very time-consuming generation of bifurcation diagrams.

The background options set here can be stored as the default settings with the **Save Defaults** command.

For more information on running Quadratic Map in the background, refer to the "MultiFinder and Memory Considerations" part of the section "Tips and Techniques."

Control Menu

Begin Iteration

The **Begin Iteration** command starts a base iteration.

Stop Iteration
Continue Iteration

The **Stop Iteration** command stops a base iteration. After being stopped, the **Continue Iteration** will continue it. When a solution is unbounded and goes to negative infinity, it stops automatically and cannot be continued.

Do One Iteration

The **Do One Iteration** command starts or continues a base iteration, whichever is appropriate, finds and graphs the next iteration, then stops the iteration.

Play Back Solutions

The **Play Back Solutions** command plays back the current iteration from the beginning.

Initial Condition

The **Initial Condition** command allows you to enter a new initial condition for the iteration. Check the (Re)Start Iteration box if you wish the iteration to begin after you click OK. Check the Clear Phase Spaces box if you wish to clear the points plotted so far in the *Phase Space* and *Embedded Phase Space* windows; doing this will in effect issue the **Clear Iterations** command from the **View** menu. The current initial condition can be made the default one with the **Save Defaults** command.

System Parameter

The **System Parameter** command allows you to change the system parameter lambda of the iteration. Check the (Re)Start Iteration box if you wish the iteration to begin after clicking OK. Changing the system parameter will eliminate all data previously generated. The current system parameter can be made the default one with the **Save Defaults** command.

Get Initial from Last

The **Get Initial from Last** command makes the last value of the iteration the new initial condition, bringing up the **Initial Condition** dialog box.

Maximum Iterations

The **Maximum Iterations** command allows you to set the maximum number of solutions a base iteration will generate. The current maximum iterations can be made the default one with the **Save Defaults** command.

View Menu

Graph Range

When you first start it up, Quadratic Map graphs data (in the *Phase Space*, *Embedded Phase Space*, and *Orbit* windows) in the following manner. For $0 \le \lambda \le 4$, Quadratic Map graphs from 0 to 1. For $\lambda > 4$, Quadratic Map graphs from 0 to $\lambda/4$. For $0 > \lambda \ge -4$, Quadratic Map graphs from –1 to 1. For $\lambda < -4$, Quadratic Map graphs from $\lambda/4$ to $|\lambda/4|$. The **Graph Range** command allows you to change the range of values graphed. For the *Phase Space* and *Embedded Phase Space* windows, the graph range given here can be overridden by magnification. The graph range set here can be made the default graph range with the **Save Defaults** command.

Iteration Speed

The **Iteration Speed** command allows you to slow down the iteration so that the graphing can be watched more easily. The dialog box this command brings up gives you a scroll bar on which you can set the speed, from slow to fast. The slowest speed corresponds to approximately one iteration every five seconds. If the fastest speed is selected, the speed is limited only by the speed of the computer. The **Save Defaults** command will remember the last speed selected as the default speed.

Phase Space

The **Phase Space** command gives you control over the display in the *Phase Space* window; refer to the discussion of that window in the section "Quadratic Map Windows" for further description of these options.

This command allows you to display a number line to graph the data, and set the number line's width (in pixels). It also allows you to turn on and off the display of the parabola the quadratic map is based on, the 45° line $x_{n+1} = x_n$, the outline of the rectangle that goes from $x_{n+1} = 0$ to $x_{n+1} = 1$ and from $x_n = 0$ to $x_n = 1$, and vertical and horizontal lines that show the path from one solution to the next, alternatively with arrows showing the direction of iteration. If you are turning on display of these latter lines, click Show from n=0 to display all the lines from the beginning of the iteration; otherwise, only lines for solutions yet to be generated will be displayed. Finally, click Same Size Axes to use symmetric axes.

The **Save Defaults** command makes the current values of the parameters set with this command the default values.

Embedded Phase Space

The **Embedded Phase Space** command gives you control over the display in the *Embedded Phase Space* window; refer to the discussion of that window in the section "Quadratic Map Windows" for further description of these options.

Most importantly, this command allows you to set the embedding of the embedded phase space, the number that determines to which past iteration you are comparing the current iteration. It also allows you to turn on and off the display of the 2^n-degree polynomial that results from composing the quadratic map a number of times equal to the embedding (all points will fall on this polynomial), the 45° line $x_{n+k} = x_n$, where k equals the embedding, and the outline of the rectangle that goes from $x_{n+k} = 0$ to $x_{n+k} = 1$ and from $x_n = 0$ to $x_n = 1$. Finally, click Same Size Axes to use symmetric axes.

The **Save Defaults** command makes the current values of the parameters set with this command the default values.

Orbit

The **Orbit** command gives you control over the display in the *Orbit* window. You can set the size (in square pixels) of the points it plots in this window, and the horizontal space (in pixels) between these points.

The **Save Defaults** command makes the current values of the parameters set with this command the default values.

Scroll Orbit

Under the default settings, the plot in the *Orbit* window clears itself and starts over when it reaches the right edge. With the **Scroll Orbit** command, you can tell Quadratic Map to scroll the *Orbit* window when it reaches the edge. This is a toggle command; successive executions of the command turn the scrolling off and on. While the scrolling has a nice effect, it tends to slow down down the whole iteration considerably, even when the *Orbit* window is closed. The **Save Defaults** command makes the current value of this option the default value.

Clear Iterations

The **Clear Iterations** command clears the data in the *Phase Space* and *Embedded Phase Space* windows, so that only subsequent iterations are displayed there. Use this command if the windows become too cluttered. To redisplay the iterations that were calculated prior to this command, choose **Show All Iterations** from this menu.

Show All Iterations

The **Show All Iterations** command undoes the effect of the **Clear Iterations** command above; all solutions that had been generated since the beginning of the iteration will be displayed in the *Phase Space* and *Embedded Phase Space* windows.

Clear Iteration Lines

When the *Phase Space* window is displaying "iteration lines," the lines that show graphically the construction of the quadratic map, the display can get quite cluttered quite quickly. The **Clear Iteration Lines** command clears these lines, so that only those from subsequent iterations are displayed. To redisplay the lines that were drawn prior to this command, choose **Show All Iteration Lines** from this menu. This command is dimmed (grayed) when iteration lines are not being displayed.

Show All Iteration Lines

The **Show All Iteration Lines** command undoes the effect of the **Clear Iterations** command above; the lines corresponding to all solutions that had been generated since the beginning of the iteration will be displayed.

Phase Space Magnification

The **Phase Space Magnification** command allows you to view, in the *Phase Space* window, any part of the entire phase space; this allows you to gain greater detail. You specify minimum and maximum values for both axes. Press any of the Auto boxes for a corresponding default value.

Click the NO MAGNIFICATION button to use the normal (unmagnified) view.

A convenient way to call up this command is by dragging around the area to be magnified in the *Phase Space* window. Doing this will bring up the **Phase Space Magnification** dialog box with appropriate endpoints.

Embed. Phase Space Magnification

The **Embed. Phase Space Magnification** command allows you to view, in the *Embedded Phase Space* window, any part of the entire embedded phase space; this allows you to gain greater detail. You specify minimum and maximum values for both axes. Press any of the Auto boxes for a corresponding default value.

Click the NO MAGNIFICATION button to use the normal (unmagnified) view.

A convenient way to call up this command is by dragging around the area to be magnified in the *Embedded Phase Space* window. Doing this will bring up the **Embed. Phase Space Magnification** dialog box with appropriate endpoints.

Special Menu

Start Bifurcation Diagram

The **Start Bifurcation Diagram** command begins the creation of the current bifurcation diagram. If the *Bifurcation Diagram* window is not open, you will be asked if you want to open the window.

Stop Bifurcation Diagram
Continue Bifurcation Diagram

While a bifurcation diagram is running, the **Stop Bifurcation Diagram** command stops it. **Continue Bifurcation Diagram** continues a bifurcation diagram that has been stopped before completion. If you issue the **Continue Bifurcation Diagram** command and the *Bifurcation Diagram* window is not open, you will be asked if you want to open the window.

Bifurcation Diagram

The **Bifurcation Diagram** command brings up the dialog box that allows you to set all the parameters of a bifurcation diagram. Please refer to the section "Types of Iteration" for a discussion of this dialog box. If you use this command to start a bifurcation diagram and the *Bifurcation Diagram* window is not open, you will be asked if you want to open the window.

Clear Bifurcation Diagram

The **Clear Bifurcation Diagram** command clears the *Bifurcation Diagram* window and resets all bifurcation diagram parameters. Because this command is destructive, you are given the option of saving the current bifurcation diagram first.

Graph in All Windows

The **Graph in All Windows** command instructs Quadratic Map to toggle whether it displays the progress of the bifurcation diagram in all the windows. In constructing the bifurcation diagram, Quadratic Map actually runs many base iterations; one can watch these iterations as the bifurcation diagram progresses. On the other hand, often one will not want to watch these iterations, as they significantly increase the time required to complete the bifurcation diagram. Use the **Save Defaults** command to make the current setting of this option the default setting.

Use Color

The **Use Color** command toggles whether the current bifurcation diagram is displayed in color. This menu item will be disabled (grayed) if the current monitor is not a color one with at least 4-bit color (16 colors) enabled. *Note*: If the monitor is set to display gray scale, at 4 bits or more, this option will be enabled, and, instead of colors, the bifurcation diagram will be displayed in shades of gray. Use the **Save Defaults** command to make the current setting of this option the default setting.

Colors

The **Colors** command allows you to specify the colors used to represent the different types of solutions in the bifurcation diagram, when color is on. Use the **Save Defaults** command to save the current color-solution relationship as the default one.

Sound Menu

NOTE: To turn off all sound, eliminate the checks next to the **x_n Tones** command and the **x_n–k Tones** command, if any, by choosing the appropriate command.

x_n Tones
x_n–k Tones

The **x_n Tones** and **x_n–k Tones** commands toggle Quadratic Map's sound. With "x_n tones" on, Quadratic Map creates, for each iteration, a sound corresponding to where the iteration falls in the graph range, as defined by the **Graph Range** command. With "x_n–k tones" on, Quadratic Map creates a sound in the same way that corresponds to the *n–k*th iteration, where *k* equals the embedding of the *Embedded Phase Space* window, as set by the **Embedded Phase Space** command. When you first start Quadratic Map, x_n tones are on, x_n–k tones off. You can save the current settings as the default settings with the **Save Defaults** command.

The frequency of the sound(s) Quadratic Map creates (for each iteration) ranges from 256 Hz to 512 Hz as the iteration ranges from the low end of the graph range to the high end. Values out of the graph range will generate frequencies out of the 256 to 512 Hz range. See the other commands from this menu for descriptions of the sounds created.

Major
Minor
Chromatic
Continuous

These four commands determine the exact frequency of sound created by Quadratic Map. The **Major** command divides the graph range, as defined by the **Graph Range** command, into seven equal areas, corresponding to the seven tones of the major scale. To find the tone for a particular value, Quadratic Map looks at which area the value falls in, and chooses the corresponding tone of the major scale. The **Minor** command acts similarly, dividing the range into areas corresponding to the seven tones of the minor scale. The **Chromatic** command divides the range into twelve areas, corresponding to the twelve tones of the chromatic scale. With the **Continuous** command, Quadratic Map creates a sound that ranges logarithmically from 256 to 512 Hz as the iteration value ranges from the beginning of the graph range to the end.

The **Save Defaults** command will make the current "frequency-finding method" the default one.

Windows Menu

After the first three commands, which are described below, the **Windows** menu lists all of Quadratic Map's windows, placing a diamond next to the windows that are open. Choosing one of these menu items brings the named window to the front, making it visible if necessary.

Close Window
Close All Windows

The **Close Window** command closes the front window. If the Option key is depressed, the command becomes **Close All Windows** and will close all open windows (holding down Option while clicking in the front window's close box will do the same thing). If there are no open windows, this command is grayed.

Arrange Windows

The **Arrange Windows** command arranges the currently-open windows in a standard format. If there are no open Quadratic Map windows, this command is grayed.

Default Configuration

The **Default Configuration** command puts Quadratic Map's windows in the configuration stored in the "Quadratic Map Defaults" file in the current system folder (see **Save Defaults** command). If there is no such file, it sizes the *Phase Space*, *Iteration Data*, *Embedded Phase Space*, and *Orbit* windows equally, and puts one into each quarter of the screen.

TIPS AND TECHNIQUES

This section contains a series of tips and techniques to make your use of Quadratic Map easier and more enjoyable.

Speeding Up the Iteration

To speed up Quadratic Map iterations, there are a few steps you can take. Turning the sound off yields a considerable increase in speed. Turning off orbit scrolling (which initially is off—see the **Scroll Orbit** command, **View** menu) also helps. Of course, if the iteration has been slowed with the **Iteration Speed** command (**View** menu), you can speed it up with that command.

Bifurcation diagrams tend to take a lot of time under any circumstances, especially with a slow Macintosh. To generate them as fast as possible, do not select the **Graph in All Windows** command and turn the sound off.

MultiFinder and Memory Considerations

Quadratic Map can run in the background under MultiFinder. The way it runs in the background is set with the **Background Operation** command from the **Edit** menu. See the description of that command for more information.

There are several considerations that should be given to memory usage with Quadratic Map under MultiFinder. First of all, if you are not familiar with memory partitioning under MultiFinder, it is set with the Get Info command in the Finder; please refer to your MultiFinder manual for more information. Quadratic Map's default memory partition is 512K. That should be sufficient to run both types of iterations, and to create base iterations of a couple thousand iterations or more.

If you are using a larger monitor or multiple monitors, you should increase the default partition size by the following amount: Multiply the number of pixels the larger monitor displays horizontally by the number of pixels it displays vertically to arrive at its resolution, its total number of pixels. In the case of multiple monitors, use the total numbers of pixels that would be displayed by a large monitor just barely encompassing all your monitors (as arranged with the Monitors cdev in the Control Panel). Subtract 200,000 from this resolution and multiply the resulting number by 0.0007. The result of this calculation represents the additional memory (in "K") that should be added to Quadratic Map's memory partition. The calculation is reproduced below:

Additional Memory = (Resolution – 200,000) * 0.0007.

In addition, if you wish to create large base iterations (> 2000 iterations), you should increase the default partition size by 40K for each 1000 iterations.

Of course, depending on the memory available in your computer, the above calculations may imply certain restrictions on the operation of Quadratic Map.

One other consideration: If you open a bifurcation diagram created and sized in a window larger than your current monitor, Quadratic Map may create "memory islands," which can cause it to be unable to fully use the available memory. Beware of this if you create bifurcation diagram files on large monitors, then open them on small monitors. By the way, the enlarged memory values given above would partially apply to such situations. Thus, even though you may be using a small Macintosh monitor, you will require much of the memory of a monitor large enough to display the entire large-sized bifurcation diagram file.

Quadratic Map and Other Programs

Quadratic Map can export data in several ways. Graphics and text can both be exported using the **Copy** command and the Clipboard. Files saved from Quadratic Map's *Iteration Data* window, being text files, can be easily shared with many programs. The **Save As** command allows you to specify the data delimiter, to ease data transfer. Bifurcation diagram files, on the other hand, use a proprietary format. The graphics in these files are stored as PICT resources; an ambitious user could use a resource editor to access these graphics and transfer them to another application, perhaps to gain greater color control, or to make color separations.

For the most part, it is not possible to import data into Quadratic Map. Technically, Quadratic Map can try to open any text file, and if that file contains data in Quadratic Map format, it will open it. If the data is not actual Quadratic Map data, however, this is likely to result in erratic performance.

Quadratic Map appears to coexist peacefully with all popular INITs, cdevs, and desk accessories. If you begin a simulation and leave the computer alone, and if you use a screen saver, the screen saver will blank the screen while Quadratic Map is creating the simulation. Quadratic Map continues to create data after the screen saver is activated. When the screen saver is deactivated, Quadratic Map will properly display all the data.

Miscellaneous

Warning or confirmation dialog boxes are generated by certain commands. For many of these commands, holding the Shift key down when executing the command will bypass the appropriate dialog box, automatically making the default response. For some of these, holding the Option key down when executing the command will effectively answer "No" to the dialog box. For instance, if you hold Shift down while executing the **Quit** command, the program will automatically save all current data before quitting, while if you hold Option down, the program will quit while neither prompting you to save data nor saving data. Shift has priority over Option; if both are held down, the Option key will be ignored. Either Shift or both Shift and Option work in this fashion with the following commands: **New**, **Open**, **Quit**, **System Parameter**, **Get Initial from Last**, **Clear Bifurcation**

Diagram, **Graph in All Windows**, and commands that start or continue bifurcation diagrams. Shift also works when magnifying in the *Phase Space*, *Embedded Phase Space*, and *Bifurcation Diagram* windows (to bypass the corresponding dialog box), and when clicking an initial condition in the *Phase Space* or *Embedded Phase Space* windows.

In any dialog box, pressing Command-letter will have the effect of clicking the "first" control (button, radio button, check box) whose title begins with that letter. The order of these controls is determined internally; in general, the OK (default) and Cancel (or analogous) buttons come first, followed by the others in order from top to bottom, left to right. In a dialog box with no text input (edit text) fields, pressing a letter without Command will click the first radio button or check box whose title begins with that letter. Command-period or Escape will cancel dialog boxes. Tab takes you forward through the text input fields; Shift-Tab takes you backward through these fields.

Command-period (□-.) will stop the current iteration, whether a base iteration or a bifurcation diagram. It will also stop the file-opening of a base iteration.

In the **Iteration Speed** dialog box, pressing the Left and Right Arrow keys moves the scroll bar's "thumb" by a small amount in the corresponding direction; pressing the Up and Down keys moves the thumb by a larger amount.

Program Problems

Bifurcation diagrams may not print properly on an Apple LaserWriter SC™ with some versions of the printer driver. If you use the latest version of the printer driver, they should print fine.

If you find any bugs or other program problems, please notify the publisher.

SELECTED ERROR MESSAGES

Most of Quadratic Map's error messages should be self-explanatory. Below are a few error messages that may require additional explanations and/or suggestions.

Not a Quadratic Map file

The text file Quadratic Map is trying to open does not have a valid Quadratic Map header (which contains the system parameter lambda). This could be because the file has nothing to do with Quadratic Map, since Quadratic Map's **Open** dialog includes all text files—refer to the discussion of the **Open** command. If the file should be a proper Quadratic Map file, you could use a word processor to look at the file, compare it to a file that Quadratic Map can open, make any required changes, save it as a text file, then open it inside Quadratic Map again.

Sorry—not enough memory to run Quadratic Map

If you are running under MultiFinder, this command means that Quadratic Map's memory partition is too small. If not, the Macintosh being used does not have sufficient memory to run Quadratic Map. Refer to the discussion of memory usage in the section "Tips and Techniques."

Unable to open entire file

One of the solutions in the file Quadratic Map is opening is not in valid format. Each solution should consist of the iteration number (an integer) and the iteration value, a floating point number. To correct this problem, inspect the file with a word processor, looking where Quadratic Map stopped loading the data, and try to find an invalid number, or an incomplete line, or something of that nature. If you are able to find and correct the problem, save it as a text file and re-open it in Quadratic Map.

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